THE FIFTH INTERNATIONAL CONGRESS OF MATHEMATICIANS, CAMBRIDGE, 1912.

THE local committee of organization was composed of Sir G. H. Darwin, president, E. W. Hobson and A. E. H. Love, secretaries, and Sir J. Larmor, treasurer. Early in the spring circulars were sent out, containing an outline of the proposed plan of procedure and an invitation to participate in the deliberations of the Congress. The date was fixed at August 21 for the opening reception, and meetings were arranged to occupy a week. Members desiring it were permitted to room in the various Colleges, Newnham College being reserved for ladies. Those who took advantage of this provision found their rooms in readiness upon their arrival in Cambridge. The secretaries also compiled a list of boarding and rooming houses in the city, together with a plan indicating the positions of the Colleges and various meeting rooms, so that all the participants were well accommodated.

The elaborate programme of entertainments and excursions which had been prepared for the Congress deserves special consideration. It consisted of four large receptions, an organ recital, and many teas, besides excursions to Ely, Oxford, Hatfield House, and many other interesting places.

The first occasion was a reception given by Sir G. H. Darwin, to meet the Vice-Chancellor, on Wednesday evening in the Combination Room and Hall of St. John's College. This was an ideal place for an introduction to the charming hospitality of Cambridge University.

On Friday evening the Chancellor and Lady Rayleigh gave a reception in the Fitzwilliam Museum. Although each social event was so perfect in its way that comparison is difficult, yet it is not unfair to say that this was the most brilliant affair of the week. The magnificent art galleries, the large number of guests, and the gorgeous academic costumes all contributed to the effect.

The members of the Congress were fortunate in the day chosen by the President to receive them at a garden party. For on Sunday there was the most perfect weather of the week. In the gardens of Christ's College the foreigners received some impression, however faint, of the possibilities of

an English August. In the evening an organ recital was given in the beautiful chapel of King's College.

The last of the large, formal receptions was held on Monday evening by the Master and Fellows of Trinity College in the Hall and Cloisters.

As regards the excursions, it was necessary to make selections, for they were more numerous than the afternoons. On Monday there were opportunities to visit Ely or the works of the Cambridge Scientific Instrument Company. The latter invitation was extended to Tuesday also. On both afternoons the Hon. Mrs. Horace Darwin served tea after the visit to the works. For Tuesday afternoon there was also an invitation to visit the University Observatory and afterwards to take tea with Professor and Mrs. Newell.

For Wednesday afternoon excursions were planned to Oxford University and Hatfield House. The latter invitation was due to the kindness of the Marquis of Salisbury.

The resourceful committee, who had made such perfect arrangements for everything on the general programme, made provision also for those ladies who had no interest in purely scientific matters. A supplementary programme was arranged, under the direction of a ladies' committee, with Lady Darwin as chairman. There were popular lectures, specially conducted visits to the various colleges, and teas given by ladies of Cambridge and by Sir Charles and Lady Waldstein.

But perhaps the most unique feature of the social arrangements was to be found in the opportunity—so freely and cordially offered to the members of the Congress—to live in these historical old buildings and to share, in a degree, the life of the University. It was characteristic of British hospitality.*

On the afternoon of the last day of the Congress a large number of members proceeded to Cayley's grave and deposited there a wreath in his memory. Funds were contributed for a silver wreath to be presented to the University as a permanent tribute of honor and respect for this great Cambridge mathematician.

At the last session of the Congress it was reported that 706 persons had been registered as in attendance, of whom 573 were active participants, representing 27 different countries.

*The above report of the social side of the Congress was prepared by Dr. ELIZABETH B. COWLEY, of Vassar College.

On the motion of Professor Mittag-Leffler it was voted to hold the next Congress in Stockholm in 1916.

Professor A. G. Webster appropriately voiced the heartfelt appreciation and thanks of the members of the Congress for the cordial hospitality lavishly showered upon them, collectively and individually, by their gracious hosts. All present expressed their hearty participation in his tribute by prolonged applause and an enthusiastic rising vote. Thereupon a few fitting words of the chairman, Sir George Darwin, brought the fifth international congress of mathematicians to a close.

The sectional work of the Congress was distributed as follows:

I. Arithmetic, Algebra, Analysis.
II. Geometry.
III*a*. Mechanics, Physical Mathematics, Astronomy.
III*b*. Economics, Actuarial Science, Statistics.
IV*a*. Philosophy and History.
IV*b*. Didactics.

The International commission on the teaching of mathematics held one separate session and three sessions jointly with Section IV*b*.

A meeting of the international committee was held at 5 P. M., Wednesday, August 21, to complete the final arrangements.

The opening meeting of the Congress was held at 10 A. M. on Thursday. Sir G. H. Darwin, President of the Cambridge Philosophical Society, spoke as follows:

Four years ago at our conference at Rome the Cambridge Philosophical Society did itself the honor of inviting the International Congress of Mathematicians to hold its next meeting at Cambridge. And now I, as president of the society, have the pleasure of making you welcome here. I shall leave it to the vice-chancellor, who will speak after me, to express the feeling of the university as a whole on this occasion, and I shall confine myself to my proper duty as the representative of our scientific society.

The science of mathematics is now so wide and is already so much specialized that it may be doubted whether there exists to-day any man fully competent to understand mathematical research in all its many diverse branches. I, at least, feel how profoundly ill equipped I am to represent our society

as regards all that vast field of knowledge which we classify as pure mathematics. I must tell you frankly that when I gaze on some of the papers written by men in this room I feel myself much in the same position as if they were written in Sanskrit.

But if there is any place in the world in which so one-sided a president of the body which has the honor to bid you welcome is not wholly out of place it is perhaps Cambridge. It is true that there have been in the past at Cambridge great pure mathematicians such as Cayley and Sylvester, but we surely may claim without undue boasting that our university has played a conspicuous part in the advance of applied mathematics. Newton was a glory to all mankind, yet we Cambridge men are proud that fate ordained that he should have been Lucasian professor here. But as regards the part played by Cambridge I refer rather to the men of the last hundred years, such as Airy, Adams, Maxwell, Stokes, Kelvin, and other lesser lights, who have marked out the lines of research in applied mathematics as studied in this university. Then too there are others such as our chancellor, Lord Rayleigh, who are happily still with us.

Up to a few weeks ago there was one man who alone of all mathematicians might have occupied the place which I hold without misgivings as to his fitness; I mean Henri Poincaré. It was at Rome just four years ago that the first dark shadow fell on us of that illness which has now terminated so fatally. You all remember the dismay which fell on us when the word passed from man to man 'Poincaré is ill.' We had hoped that we might again have heard from his mouth some such luminous address as that which he gave at Rome; but it was not to be, and the loss of France in his death affects the whole world.

It was in 1900 that, as president of the Royal Astronomical Society, I had the privilege of handing to Poincaré the medal of the society, and I then attempted to give an appreciation of his work on the theory of the tides, on figures of equilibrium of rotating fluid, and on the problem of the three bodies. Again in the preface to the third volume of my collected papers I ventured to describe him as my patron saint as regards the papers contained in that volume. It brings vividly home to me how great a man he was when I reflect that to one incompetent to appreciate fully one half of his work he yet appears as a star of the first magnitude.

It affords an interesting study to attempt to analyze the difference in the textures of the minds of pure and applied mathematicians. I think that I shall not be doing wrong to the reputation of the psychologists of half a century ago when I say that they thought that when they had successfully analyzed the way in which their own minds work they had solved the problem before them. But it was Sir Francis Galton who showed that such a view is erroneous. He pointed out that for many men visual images form the most potent apparatus of thought, but for others this is not the case. Such visual images are often quaint and illogical, being probably often founded on infantile impressions, but they form the wheels of the clockwork of many minds. The pure geometrician must be a man who is endowed with great powers of visualization, and this view is confirmed by my recollection of the difficulty of attaining to clear conceptions of the geometry of space until practice in the art of visualization had enabled one to picture clearly the relationship of lines and surfaces to one another. The pure analyst probably relies far less on visual images, or at least his pictures are not of a geometrical character. I suspect that the mathematician will drift naturally to one branch or another of our science according to the texture of his mind and the nature of the mechanism by which he works.

I wish Galton, who died but recently, could have been here to collect from the great mathematicians now assembled an introspective account of the way in which their minds work. One would like to know whether students of the theory of groups picture to themselves little groups of dots, or are they sheep grazing in a field. Do those who work at the theory of numbers associate color, or good or bad characters, with the lower ordinal numbers and what are the shapes of the curves in which the successive numbers are arranged? What I have just said will appear pure nonsense to some in this room, others will be recalling what they see, and perhaps some will now for the first time be conscious of their own visual images.

The minds of pure and applied mathematicians probably also tend to differ from one another in the sense of esthetic beauty. Poincaré has well remarked in his Science et Méthode (page 57):

'On peut s'étonner de voir invoquer la sensibilité à propos de démonstrations mathématiques qui, semble-t-il, ne peuvent

intéresser que l'intelligence. Ce serait oublier le sentiment de la beauté mathématique, de l'harmonie des nombres et des formes, de l'élegance géometrique. C'est un vrai sentiment esthétique que tous les vrais mathématiciens connaissent. Et c'est bien là de la sensibilité.'

And again he writes:

'Les combinaisons utiles, ce sont précisément les plus belles, je veux dire celles qui peuvent le mieux charmer cette sensibilité spéciale que tous les mathématiciens connaissent, mais que les profanes ignorent au point qu'ils sont souvent tentés d'en sourire.'

Of course there is every gradation from one class of mind to the other, and in some the esthetic sense is dominant and in others subordinate.

In this connection I would remark on the extraordinary psychological interest of Poincaré's account, in the chapter from which I have already quoted, of the manner in which he proceeded in attacking a mathematical problem. He describes the unconscious working of the mind, so that his conclusions appeared to his conscious self as revelations from another world. I suspect that we have all been aware of something of the same sort, and like Poincaré have also found that the revelations were not always to be trusted.

Both the pure and applied mathematicians are in search of truth, but the former seeks truth in itself and the latter truths about the universe in which we live. To some men abstract truth has the greater charm, to others the interest in our universe is dominant. In both fields there is room for indefinite advance, but while in pure mathematics every new discovery is a gain, in applied mathematics it is not always easy to find the direction in which progress can be made, because the selection of the conditions essential to the problem presents a preliminary task, and afterwards there arise the purely mathematical difficulties. Thus it appears to me at least, that it is easier to find a field for advantageous research in pure than in applied mathematics. Of course if we regard an investigation in applied mathematics as an exercise in analysis the correct selection of the essential conditions is immaterial, but if the choice has been wrong the results lose almost all their interest. I may illustrate what I mean by reference to Lord Kelvin's celebrated investigation as to the cooling of the earth. He was not and could not be aware of the radio-activity of the

materials of which the earth is formed, and I think it is now generally acknowledged that the conclusions which he deduced as to the age of the earth cannot be maintained; yet the mathematical investigation remains intact.

The appropriate formulation of the problem to be solved is one of the greatest difficulties which beset the applied mathematician, and when he has attained to a true insight but too often there remains the fact that his problem is beyond the reach of mathematical solution. To the layman the problem of the three bodies seems so simple that he is surprised to learn that it cannot be solved completely, and yet we know what prodigies of mathematical skill have been bestowed on it. My own work on the subject cannot be said to involve any such skill at all, unless indeed you describe as skill the procedure of a housebreaker who blows in a safe-door with dynamite instead of picking the lock. It is thus by brute force that this tantalizing problem has been compelled to give up some few of its secrets, and great as has been the labor involved I think it has been worth while. Perhaps this work too has done something to encourage others, such as Störmer, to similar tasks as in the computation of the orbits of electrons in the neighborhood of the earth, thus affording an explanation of some of the phenomena of the aurora borealis. To put at their lowest the claims of this clumsy method, which may almost excite the derision of the pure mathematician, it has served to throw light on the celebrated generalizations of Hill and Poincaré.

I appeal then for mercy to the applied mathematician and would ask you to consider in a kindly spirit the difficulties under which he labors. If our methods are often wanting in elegance and do but little to satisfy that esthetic sense of which I spoke before, yet they are honest attempts to unravel the secrets of the universe in which we live.

We are met here to consider mathematical science in all its branches. Specialization has become a necessity of modern work and the intercourse which will take place between us in the course of this week will serve to promote some measure of comprehension of the work which is being carried on in other fields than our own. The papers and lectures which you will hear will serve towards this end, but perhaps the personal conversations outside the regular meetings may prove even more useful.

Mr. R. F. Scott, Vice-Chancellor of the University of Cambridge, spoke as follows:

Gentlemen: It is my privilege to-day on behalf of the University of Cambridge and its Colleges to offer to members of the Congress a hearty welcome from the resident body.

Sir George Darwin has dwelt on the more serious aspects of the meeting and work of the Congress; may I express the hope that it will also have its lighter and more personal side? That we shall all have the privilege and pleasure of making the personal acquaintance of many well known to us both by name and by fame, and that those of our visitors who are not familiar with the college life of Oxford and Cambridge will learn something of a feature so distinctive of the two ancient English universities. If the Congress comes at a time when it is not possible to see the great body of our students either at work or at play, the choice of date at least renders it possible that many of our visitors may enjoy for a time that collegiate life which has so many attractions.

I see that one of the Sections of the Congress deals with historical and didactical questions. Those members of the Congress who are interested in these subjects will have an opportunity of learning on the spot something of our methods in Cambridge, and of the history of our chief mathematical examination, the mathematical tripos, and of its influence on the study and progress of mathematics both in Cambridge and Great Britain.

The researches of Dr. Venn seem to point to the fact that until it was altered at a very recent date the mathematical tripos represented something like the oldest example in Europe of a competitive examination with an order of merit. Those who are interested in such matters of history will find much to interest them in Mr. Rouse Ball's History of the Study of Mathematics at Cambridge.

The subject is to me, and I hope to others, an interesting one. There can be no doubt that the examination and the preparation for it has had a profound influence on mathematical studies at Cambridge.

Many Cambridge mathematicians, as the names given by Sir George Darwin testify, studied mathematics for its own sake and with the view of extending the boundaries of knowledge. Many others, probably the great majority, studied mathematics with their eyes fixed upon the mathematical

tripos, with the view in the first place of being examined and afterwards of acting as examiner in it.

The tendency at Cambridge has been to give great minuteness to the study of any particular branch of mathematics, to stimulate the invention of what we call "problems," examples of more general theories. If I may borrow a simile from the study of literature the tendency was to produce critics and editors rather than authors or men of letters, followers rather than investigators. The effect must I think be obvious to any one who compares Cambridge text books and treatises with those of the continental schools of mathematics. I may illustrate what I mean by referring to the Mathematical Problems of the late Mr. Joseph Wolstenholme, a form of work I believe without a parallel in the mathematical literature of other nations. The fashion is fading away, but while you are in Cambridge I commend it to your notice.

Professor E. W. Hobson, senior secretary of the Organizing Committee, stated that the number of persons who had joined the Congress up to 10 P. M. on Wednesday, August 21, was 670, the number of representatives of different countries being as follows: Austria 19, Belgium 4, Brazil 4, Bulgaria 1, Canada 4, Chili 1, Denmark 5, Egypt 2, France 52, Germany 70, Great Britain 250, Greece 5, Holland 9, Hungary 19, India 3, Italy 38, Japan 3, Mexico 1, Norway 4, Portugal 3, Roumania 5, Russia 38, Servia 1, Spain 25, Sweden 13, Switzerland 9, United States 82. He also called the attention of the members of the Congress to the exhibition of books, models and machines (chiefly calculating machines) arranged in two rooms of the Cavendish Laboratory.

The first general meeting of the Congress was held at 2.30 P. M.

On the motion of Professor Mittag-Leffler, seconded by Professor Enriques, Sir G. H. Darwin was elected president of the Congress.

On the motion of the president it was agreed that Lord Rayleigh be made honorary president of the Congress.

General Lectures.

Eight general lectures were provided for the afternoons as follows:

1. ENRIQUES, F., Bologna: "Il significato della critica dei principii nello sviluppo delle matematiche."
2. BROWN, E. W., New Haven: "Periodicity in the solar system."
3. LANDAU, E., Göttingen: "Gelöste und ungelöste Probleme aus der Theorie der Primzahlverteilung und der Riemann'schen Zetafunktion."
4. GALITZIN, B., St. Petersburg: "The principles of instrumental seismology."
5. BOREL, E., Paris: "Définition et domaine d'existence des fonctions monogènes uniformes."
6. WHITE, W. H., London: "The place of mathematics in engineering practice."
7. BÔCHER, M., Cambridge, Mass.: "Boundary problems in one dimension."
8. LARMOR, J., Cambridge, Eng.: "The dynamics of radiation."

Abstracts of the lectures follow below.

1. PROFESSOR F. ENRIQUES.

An analysis of fundamental principles is the order of the day among contemporary mathematicians. The profound analysis of the concepts of limit and of function, the investigations having for point of departure the theory of parallels and of non-euclidean geometry, the more recent ones concerning the foundations of geometry and of analysis situs, the developments on varieties of several dimensions, on transformations and their groups, these and other problems have given rise to a series of questions lying at the very roots of the mathematical edifice, and of philosophic thought.

History furnishes instructive information on this point. The analysis of principles is not, properly speaking, a new phenomenon, characterizing our time; it is, on the contrary, an essential part of the elaboration of the concepts which in every age accompanies the progress of the science and of its application. The universally admired perfection of the work of Euclid is revealed to the historian as the natural product of a long criticism which was developed in the constructive period of rational geometry, from Pythagoras to Eudoxus. Then commenced to appear the signification of those methods and principles by means of which the Greeks themselves attempted

to interpret and to conquer the paradoxes concerning infinity. These are the same difficulties which reappeared at the time the infinitesimal calculus was founded, and are now again asserting themselves in the most refined analysis.

The foundation of mensuration first raised the question of geometric continuity, and the relation between the side and the diagonal of a square presented an insurmountable difficulty to the Pythagorean school. The contributions of Eudoxus and of Archimedes not only showed this question in its true light, but also contained the correct elements of the calculus.

The second period was ushered in by Galileo, Fermat, Descartes, Wallis, Roberval, etc., and was soon followed by the epoch-making advances of Newton and of Leibniz in the concept of an analytic limit and of the derivative.

The foundations of their methods and the correctness of their results were soon to be justified and clarified by the researches of Lagrange and of Carnot. These same ideas have again been considered by Cauchy, Riemann, Weierstrass, and have served as basis of the theory of the calculus of variations which has received so much attention during the last few years.

Another line of development is marked by the concept of an arbitrary function and the modern elaboration of the concept of infinity, inaugurated by Dirichlet and by Abel and Jacobi, respectively, later to be refined by Riemann, Weierstrass, Dini, Cantor, Dedekind, Veronese, Poincaré and Hilbert. Throughout all this growth, the fundamental idea of geometric continuity, as understood today, is still that of Eudoxus and of Archimedes.

Corresponding to the growth of the concepts already mentioned, another point of view was being presented, which may be called intensive mathematics, the establishing of the line of demarcation between intuition and of logical proof. The study of irrational numbers and of algebraic equations together with their natural extension to that of differential equations, to integral equations, and their various combinations, lead to a properly intense study of the principles of mathematics. These studies have found their present culmination in the theory of algebraic functions, the foundations of geometry and in certain developments of algebra, illustrated by the Clebsch-Noether theory and by recent investigations of Picard and of Poincaré.

The criticism of principles forms an integral part of the history of the development of mathematics, as well from the extensive as from the intensive point of view.

Even the exaggerations to which modern criticism may lead have served to convey a correct idea of the value of logic by contrast, by comparing with the results obtained by other processes. The importance of the concept of logic thus brought to light has given rise to the philosophy called pragmatism.

Those who oppose this concept of our physical experiences idealized wish to see in mathematics only an instrument to be used in natural sciences. But such a view (which the exaggerations of logical pragmatism could, by reaction, suggest to certain minds) would have the effect of singularly restricting the field of mathematics.

2. Professor E. W. Brown.

After pointing out the different branches into which celestial mechanics has become divided during the last thirty years, the lecturer dwelt at some length on the periods of the oscillations by which astronomers have usually expressed the motions of the bodies of the solar system. These oscillations are of short-period, long-period, secular, and librational. It was then shown how, by a consideration of these periods, we can make certain that the present theories of the moon and planets shall fully represent the motions of those bodies during the limited time for which observations exist.

The theories for the asteroids are much more difficult. In them, approximate or exact commensurability between the mean period of revolution of the asteroid round the sun and that of Jupiter plays an important part. Exact commensurability, if it exists, produces oscillations which are generally known as librations, and the mathematical theory of these is still in a very incomplete state. So far as we know, there is no reason why librating asteroids should not exist. But there are notable deficiencies in the libration regions. A similar state of things occurs in the ring of Saturn. On the other hand, we have librations in the satellite systems of both Jupiter and Saturn. It was suggested that the presence of a libration region in the problem of three bodies limits the range of stable orbits and that a still greater limitation of the range is produced by the presence of a fourth body, e. g., of Saturn, when the asteroids are under consideration.

The periodic deviations of the moon from its theoretical orbit were briefly dealt with and methods for exploring their sources were mentioned. Professor Brown concluded his lecture with a warm tribute of respect to the memory of Henri Poincaré.

3. Professor E. Landau.

Familiarity with the theory of numbers is by no means general among mathematicians; in particular, the difficulties of the analytic theory of numbers have not been attractive, so that but few are familiar with the elegant results of the theory.

After stating the problem of the lecture, a rapid resumé of preceding results and methods was given, and a concise statement of several new results, particularly those of Littlewood, Bohr, and Landau, was added.

Four definite questions were put, the solutions of which were considered as impossible in the present state of the science.

1. Does the function $u^2 + 1$, u an integer, represent an infinite number of primes?

2. Does the equation $m = p + p'$ have a solution in prime numbers for every even value of m?

3. Has the equation $2 = p - p'$ an infinite number of prime solutions?

4. Does at least one prime number lie between n^2 and $(n + 1)^2$ for every integral value of n?

4. Prince B. Galitzin.

The rapid advance of seismology in the last twenty years is mainly due to the fact that this new scientific discipline has adopted pure physical methods of research based upon instrumental observations. Instrumental seismology or seismometry, in devising its instruments of research, stands in close connection with theoretical mechanics, so intimately linked with pure mathematics. The propagation of seismic disturbances, generated in the focus of an earthquake, is nothing but a problem in the theory of elasticity. According to the latter two types of seismic waves are propagated through the interior of the earth, viz., longitudinal and horizontal or transverse waves, the velocities of which in the upper strata of the earth's crust are respectively equal to 7.17 and 4.01

kilometers per second. From the difference of time of arrival of these characteristic disturbances at a given point, the epicentral distance can be deduced.

The general equations of the theory of elasticity lead, as Lord Rayleigh and Lamb have shown, to the conclusion that another class of waves, viz., gravitational or long waves, are propagated with a constant velocity 3.5 along the surface of the earth. The arrival of these waves constitutes the beginning of the maximal phase of a seismogram. These theoretical results are confirmed in their general outlines by direct observations.

Instead of studying the seismic waves in the interior of the earth, it is more convenient to consider the corresponding seismic rays, which travel along characteristic paths. If the law which gives the relationship between velocity and depth were known, one could easily express the time of traveling from the focus to the point of observation as well as the corresponding epicentral distance as function of the depth of the focus and the angle of emergence of the seismic rays. This would give the form of the so-called vertical hodograph curve. By recording the problem, that is, plotting the form of the hodograph from direct observations, it is possible to get at some conclusions concerning the interior construction of the earth. This is the procedure adopted by Wiechert and his students.

Whereas in the boundaries of the epicentral area nearly every earthquake is characterized by several more or less intensive shocks, separated by intervals of calm, the whole disturbance seldom lasting more than a few minutes, seismic records at distant stations reveal a continuous and very prolonged motion of the ground. This stretching out of seismograms at distant stations may be attributed to inner reflections and refractions of the rays, to the release of normal vibrations of the earth's outer shell and to seimic dispersion. The problem of seismic dispersion has not been thoroughly investigated.

The proper way to treat the different problems of the propagation of seismic waves would be to consider the different layers of the earth not as an isotropic, but only as a transversely isotropic medium. This is the method followed by Rutzki, which has led him to obtain very interesting results, but the problem is a very difficult one.

Instrumental observations, which form the basis of modern

seismology, are always capable of giving, after a careful analysis of the record, the true movement of the element of the earth's surface. As there are six different possible displacements, three translations and three rotations, the problem of solving the fundamental problem of seismology, which is to find the true motion of the ground as function of the time, requires six different seismographs. The rotation about a horizontal axis corresponds to a tilting of the ground. For distant earthquakes this tilting is so small that translations alone are studied at present. To study the true movement of a particle of the earth's surface, different types of seismographs are used, all of which are based upon the principle of inertia, that is, upon the principle of the so-called steady point.

As it is necessary, in order to obtain a high magnification, that the proper period of oscillation of the seismograph should be long, horizontal pendulums are most frequently used for the study of horizontal displacements. There are different types of horizontal pendulums, those with two pivots (Rebew-Daschwitz), those with one pivot or flat spring (Molne, Omsri-Bosch, Mainka, Galitzin), or with no pivots at all and where the heavy mass is supported by two inclined wires (Zöllner, suspension). These latter pendulums introduce no friction and are of a very sensitive nature. They are used at all Russian first class seismic observatories. Wiechert's improved astatic pendulum is also much used.

To study the vertical component special vertical seismographs are used, based upon the use of springs (Vicatiori, Wiechert, Galitzin).

The movements of the seismograph needle are registered either mechanically by means of smoked paper or optically by means of a reflected ray of light. The former method introduces a good deal of friction, especially when magnifying levers are used, and are in effect troublesome. To avoid a very high magnification and avoid all friction, galvanometric registration is very advantageous. It consists in fixing to the pendulum boom several induction coils between the poles of a pair of horse-shoe magnets. When the boom is set in motion electric currents are generated in the coils and are transmitted to a very sensitive dead-beat galvanometer, whose movements are directly recorded optically. This method of registration offers many advantages, the principal one of which is that it admits of registration at a distance.

In order to obtain trustworthy records, from which one may deduce the true motion of the ground, every seismograph ought to be highly damped. This is at present a well-known and adopted axiom, but it has not as yet found its practical realization in all countries. At present air and oil damping are used, but the most simple, practical and theoretically sound mode of damping is that of magnetic damping, which is being introduced at all Russian seismic observatories.

The fundamental problem of seismology, that is, the determination of the true motion of the ground for a given interval of time, offers great difficulties, where the aid of mathematicians, as in many other fields of modern science, is much needed.

The reading of seismograms obtained by galvanometric registration from aperiodic seismographs enables us to attack different problems, some of which are of great practical importance for modern seismology.

From the readings of the aperiodic horizontal pendulums at right angles to each other at the beginning of the first phase, when the first longitudinal waves strike the ground, one can deduce the true azimuth of the epicentral; by combining this result with the epicentral distance, deduced from the difference of time of arrival of the first horizontal and longitudinal waves, we can locate the position of an epicenter from observations made at one station only. This is the method exclusively used at present at the seismic observatory at Pulkava. Taking into consideration also the displacement of the vertical seismograph we can obtain the visible angle of emergence of the seismic rays, which is a very important element for the study of the paths of the seismic rays in the interior of the earth.

The instruments devised up to the present to study the tilting of the ground independently of all displacements have not led to practical results, but a double differential pendulum with galvanometric registration seems to be a well adapted instrument for that purpose, as experiments made with a variable platform have shown that it is entirely uninfluenced by any displacements at all, however irregular they may be.

The study of extraseismic movements in the epicentral area itself may be based upon observations of excited blocks.

5. Professor E. Borel.

Cauchy insisted on various occasions on the importance of monogeneity. If we consider an elementary function obtained by simple operations on z (polynomials in z, series in z everywhere convergent representing the exponential series, trigonometric functions, etc.) and if, for such a function $F(z)$ we calculate the ratio

$$\frac{F(z+\Delta z)-F(z)}{\Delta z},$$

then if this ratio tends towards a definite limit, as Δz approaches zero, independently of the manner in which Δz approaches zero, Cauchy calls the function $F(z)$ monogenic.

It is by aid of the fundamental theorem of Cauchy,

$$f(\zeta)=\frac{1}{2\pi i}\int_c \frac{f(z)dz}{z-\zeta},$$

that we demonstrate that homogeneity in a domain W implies analyticity within the domain.

We must also have recourse to this theorem to study functions which are monogenic in a domain other than W; it will be convenient, in order to reason in a most general manner concerning the deductions from possible definitions of these functions, to consider them as defined according to Riemann, that is, admitting that we know nothing of such a function, except that it is monogenic. It is also necessary to show that the theory thus constructed is not meaningless, by furnishing actual examples of functions defined in a manner that is not ideal, but explicit.

Functions which are monogenic but not analytic possess the most important properties of analytic functions; in particular, for a region C, the existence of the first derivative implies the existence of derivatives of all orders.

The transformation of Cauchy's integral into a double integral corresponds to the hypothesis, physically natural enough, that there are no infinite masses, but only singular regions in which the density can be very high, or, if one prefers, "spheres" of finite action attached to each singular point. Monogenic non-analytic functions correspond to the case in which the singular regions are at once extremely small

and extremely numerous. I pointed out some time ago that by means of certain arithmetic dispositions of such singular regions, the lines of continuity which touch these regions without penetrating them can be such that their properties are intimately connected with the numerical simplicity of their coefficients of direction. I do not know whether some analyst more skilled than I may some day draw from these somewhat vague considerations consequences of interest to physicists, but it is impossible for me to ignore the fact that I have often been guided by the analogies of the new theory with the theory of molecular physics, to the progress of which this city and this country have so powerfully contributed.

6. Sir W. H. White.

The foundations of modern engineering have been laid on mathematics and physical science; the practice of engineering is now governed by scientific methods applied to the analysis of experience and the results of experimental research. Engineering has been defined as "the art of directing the great sources of power in nature for the use and convenience of man." An adequate acquaintance with the laws of nature, and obedience to those laws, are essential to the full utilization of these sources of power. It is now universally recognized that the educated engineer must possess a good knowledge of the sciences which bear upon his professional duties, in combination with thorough practical training and experience in actual engineering work. Neither side of his education can be neglected without hampering him seriously, especially when he has to go beyond precedent and face new problems. Of these sciences, the mathematical is undoubtedly of the greatest importance to engineers. The range and character of mathematical knowledge which can be considered adequate are gradually being agreed upon as experience is enlarged; and present ideas are embodied in the courses of study prescribed in the calendars of schools of engineering. Absolute identity in the courses of study and the standards laid down for degrees in engineering has not been attained, but the approach thereto has already been considerable, and the movement will undoubtedly continue in the same direction.

The preponderance of opinion amongst engineers now favors the teaching to students of engineering of science generally,

and of mathematics in particular, being undertaken by recognized authorities in the several branches, and on lines which shall ensure greater breadth of view and fuller capability for dealing with new problems arising in their professional work, than can be secured by means of special courses of instruction arranged for students of engineering as a class apart. Whatever branch of engineering a man may select for his individual practice, he must need a fundamental knowledge of mathematics, and in some branches in order to do his work well he will require to add considerably to the mathematical knowledge which is sufficient for a degree. As time passes, the mathematician and the practicing engineer have come to understand one another better, and to be mutually helpful. While engineers as a class cannot claim to have made many important or original contributions to mathematical science, some men trained as engineers have done notable work of a mathematical character. The names of Rankine, William Froude, and John Hopkinson, amongst British engineers, also hold an honored place in mathematics. Mathematicians of eminence have spent their lives in the tuition of engineers, and in that way have greatly influenced the practice of engineering; but while they have necessarily become familiar with the problems of engineering as a consequence of their connection therewith, they have not accomplished much actual engineering work, and none of it has been of first importance. Speaking broadly there is an abiding distinction between mathematicians and engineers. Mathematicians regard engineering chiefly from the scientific point of view, and are primarily concerned with the bearing of mathematics on engineering practice, the construction of theories and the framing of useful rules. Engineers, even when well equipped with mathematical knowledge, are primarily devoted to the design and construction of efficient and durable works; their main object being to secure the best possible association of efficiency and economy, and so to achieve practical and commercial success. There is evidently room for both classes, and their collaboration in modern times has produced wonderful results.

The proper use of mathematics in engineering practice is now generally agreed to include the following steps. First comes the development of a mathematical theory based on assumptions which are thought to embody and to represent

conditions disclosed by past practice and observation. Frequently these theoretical investigations give rise to valuable suggestions for further observations or experimental investigations. Mathematical analysis must be applied to the results of observation and experiment; and as a result amendments or extensions are made of the original mathematical theory. Useful rules are also devised, in many instances, which serve for guidance in the future practice of engineers. Formerly it was thought by men of science that purely mathematical investigation and reasoning would do all that was required for the guidance of engineering practice; now it is admitted that such investigations will not suffice, and that the chief services which can be rendered to engineering by mathematicians will consist in the suggestion of the best directions and methods for experimental research, the conduct of observations on the behavior of existing works, the establishment of general principles based on analysis of experience, and the framing of practical rules embodying scientific principles.

The contrast between present and past methods can be illustrated by comparing investigations made during the eighteenth century, into the behavior of ships amongst waves by Daniel Bernoulli who won the prize offered by the Royal French Academy of Science in 1757, and work done by William Froude a century later in connection with the same subjects. Bernoulli was the greater mathematician but had only a small knowledge of the sea and of ships. His memoir was a mathematical treatise; his practical rules, although deduced from mathematical investigations which were themselves correct, depended upon certain fundamental assumptions which did not correctly represent either the phenomena of wave motion or the causes producing and limiting the rolling oscillations of ships. Bernoulli realized and dwelt upon the need for further experiment and observation and showed remarkable insight into what was needed, but the fact remains that he neither made such experiments himself nor was able to induce others to make them. As a consequence, his practical rules for the guidance of naval architects were incorrect and would have produced mischievous results if they had been applied in practice. William Froude was a trained engineer who had a good knowledge of mathematics and a mathematical mind. His acquaintance with the sea and ships was considerable;

his skill as an experimentalist was remarkable, and he was fortunate enough to secure the support of the Admiralty, through the constructive department. He thus obtained the services of the officers of the Royal Navy in making a long series of accurate and detailed observations of the characteristic features of ocean waves as well as the rolling of ships amongst waves or in still water. In this way, starting with the formulation of a mathematical theory of wave motion, and of a theory for unresisted rolling in still water and amongst waves, Froude added corrections based on experimental research and succeeded eventually in devising methods by means of which naval architects can make close approximations to the probable behavior of ships of new design when exposed to the action of waves, either forming a regular series or constituting an irregular sea. In these approximations allowance can be made for the effect of water resistance to the rolling motion—a most important factor in the problem which could not be dealt with until experimental research had been made, and results had been subjected to mathematical analysis. In addition, Froude laid down certain practical rules for the guidance of naval architects, and the application of these rules has been shown by long experience to favor the steadiness—that is to say the comparative freedom from rolling—of ships designed in accordance with these rules. In short a problem which had proved too difficult when attacked by Daniel Bernoulli in purely mathematical fashion was practically solved a century later by Froude, who employed a combination of mathematical treatment and experimental research.

Another example of the contrast between earlier and present methods is to be found in the treatment of the resistance offered by water to the onward motion of ships. From an early date mathematicians have been attracted to this subject, and many attempts were made to frame mathematical theories. When steam propulsion for ships was introduced, the matter became of great practical importance, because it was necessary to make estimates for the engine power required to drive a ship at the desired speed. In making such estimates it was necessary to approximate to the value of the water resistance at that speed, although the required engine power was also influenced by the efficiency of the propelling apparatus and propellers. In addition it was obvious that the water resistance to the motion of a ship when she was driven by her

propellers at a given speed would be in excess of the resistance experienced if she were towed at the same speed, and there was no exact knowledge in regard to that increment of resistance. The earlier mathematical theories of resistance prove to be of little or no service, and they were based on erroneous and incomplete assumptions. Rankine devised a "stream line" theory which was superior to its predecessors, but it also for a time had no effect on the practice of naval architects. William Froude, adopting this stream line theory, dealt separately with frictional resistance, and devised a "law of comparison" at corresponding speeds by which from the "residual resistance" of models—exclusive of friction—it became possible to estimate the corresponding residual resistance for ships of similar forms. At first he stood alone in advocating these views, but subsequent experience during forty years has demonstrated their soundness. Experimental tanks for testing models of ships, such as Froude introduced, are now established in all maritime countries, and the results obtained therein are of enormous value to the designing of steam ships. In regard to the selection of the forms of ships naval architects are now able to proceed with practical certainty; but in connection with the design of screw propellers even after model experiments have been made with alternative forms of screws, there is still great uncertainty, and dependence upon the results obtained on "progressive" speed trials of ships is still of the greatest service. As yet the "law of comparison" between model screws and full-sized screws has not been determined accurately. The condition of the water in which screws act, as influenced by the advance of a ship and her frictional wake, the phenomena attending the passage of the water through a screw, and the impression thereon of sternward motion from which results the thrust of the propeller, the effect upon that thrust of variations in the forms and areas of the blades of screw propellers, and the causes of "cavitation," all form subjects demanding further investigation. In these cases, the only hope of finding solutions lies in the association of experimental research with mathematical analysis. There have been very many mathematical theories of the action of screw propellers, but none of these has provided the means for dealing practically with the problems of propeller design, and there is no hope that any purely mathematical investigation ever will do so, because the conditions

which should be included in the fundamental equations are complex and to a great extent undetermined.

In connection with other branches of engineering, model experiments have also proved effective. Examples are to be found in connection with the estimates for wind pressure on complicated engineering structures—such as girder or cantilever bridges. Experimental methods are also being applied with great advantage to the study of aeronautics and the problems of flight.

The association of the mathematical analysis of past experience with designs for new engineering works of all kinds is both necessary and fruitful of benefits. A striking example of this procedure is to be found in connection with the structural arrangements of ships of unprecedented size, which have to be propelled at high speeds through the roughest seas, to carry heavy loads, to be exposed to great and rapid changes in the distribution of weight and buoyancy, and to be subjected simultaneously to rolling, pitching and heavy motions, as well as to blows of the sea. In such a case, purely mathematical investigation would be useless: the scientific interpretation of past experience and the comparison of results of calculations based on reasonable hypotheses for ships which have seen service with similar results of calculations for ships of new design are the only means which can furnish guidance.

In the past, the association of mathematicians and engineers has done much towards securing remarkable advances made in engineering practice; and in future it may be anticipated that still greater results will be attained now that the true place of mathematicians in that practice is better understood and utilized.

7. Professor Maxime Bôcher.

In this lecture a systematic discussion was attempted of the chief advances, both in methods and results, in the most central portions of the field in question; and in doing this it was possible at some points to unify and systematize the subject beyond the point so far secured in the literature. The lecture was confined almost entirely to *linear* boundary problems, that is, to the question of solving a linear differential equation subject to linear boundary conditions.

Sir Joseph Larmor.

The essential characteristic of an electrodynamic system is the existence of the correlated fields, electric and magnetic, which occupy the space surrounding the central body, and which are an essential part of the system; to the presence of this pervading ætherial field, intrinsic to the system, all other systems situated in that space have to adapt themselves. When a material electric system is disturbed, its electrodynamic field becomes modified, by a process which consists in propagation of change outward, after the manner of radiation, from the disturbance of electrons that is occurring in the core. The practical problems of electrodynamics are of this nature—how does the modified field of force, transmitted through the æther from a disturbed electric system, and thus established in the space around and alongside the neighboring conductors which alone are amenable to our observation, penetrate into these conductors and thereby set up electric disturbance in them also? and how does the field emitted in turn by these new disturbances interact with the original exciting field and with its core?

The idea—introduced by Faraday, developed into precision by Maxwell, expounded and illustrated in various ways by Heaviside, Poynting, Hertz—of radiant fields of force, in which all the material electric circuits are immersed, and by which all currents and electric distributions are dominated, is the root of the modern exact analysis of all electric activity.

From these postulates it is possible to understand the guidance of electric radiation by wires, the general theory of pressure exerted by waves, and to arrive at the conclusion that the pressure of radiation arises wholly from momentum carried along by the waves. The idea can be illustrated by a stretched string, but the analogy is not complete.

The next topic considered was that of momentum in æthereal fields, followed by a discussion of frictional resistance to the motion of a radiating body.

Virgil Snyder.

Cambridge, Eng.,
September, 1912.

THE FIFTH INTERNATIONAL CONGRESS OF MATHEMATICIANS. SECTION I: ARITHMETIC, ALGEBRA, ANALYSIS.

PROFESSORS E. B. Elliott, E. Landau, É. Borel, E. H. Moore, and H. von Koch acted as chairmen. Following is the list of papers, with abstracts, so far as they have been furnished by the writers.

HARDY, G. H. and LITTLEWOOD, J. E.: "Some problems of Diophantine approximation."

DRACH, J.: "Sur l'intégration logique des équations différentielles."

MOORE, E. H.: "On the fundamental functional operation of a general theory of integral equations."

In this paper Professor Moore states the principal general theorems involving the functional operation J of the system Σ_5 of terms and postulates defined in his paper in the April, 1912, number of the BULLETIN. On the basis of these theorems one may, along the lines of Fredholm, Plemelj, Hilbert, E. Schmidt, Goursat, Landsberg and I. Schur, develop a general theory of linear integral equations which contains as instances current theories of the equations I, II_n, III, IV defined in the previous paper.

BERNSTEIN, S.: "Sur les recherches récentes relatives à la meilleure approximation des fonctions continues par des polynomes de degré donné."

SILBERSTEIN, L.: "Some applications of quaternions."

In the absence of Professor Silberstein this paper was read by title.

MACFARLANE, A.: "On vector analysis as generalized algebra."

The ordinary algebraic quantities may be taken as numbers or as vectors having a common direction. The corresponding elements of the generalized algebra are vectors

having any direction in space. In a sum the vectors are successive either in a line or in a cycle; their resultant is not the full equivalent of the complex. The paper investigates the consequent changes in the rules and processes of algebra including the general form of the binomial theorem. For infinitesimal successive vectors the complex is a curve; the rules for differentiation and integration are investigated and the generalized form of Leibniz' theorem is obtained, as well as the generalized formulas for the differential and ∇ of the nth order. This may be said to be the geometric part of generalized algebra. The trigonometric or transcendental part is analogous. The directed logarithm of the angle is a Hamiltonian vector, and the algebra of these logarithms is very largely analogous to that for the line vectors.

JOURDAIN, P. E. B.: "The values that certain analytic functions can take."

In the absence of Professor Jourdain, his paper was presented by Professor Hardy.

KÜRSCHÁK, J.: "Limesbildung und allgemeine Körpertheorie."

Study of K. Hensel's p-adic numbers led the author to introduce a new concept, a generalization of absolute value. To every element a of a field K let there be assigned a real number $\| a \|$ satisfying the conditions: (1) $\| 0 \| = 0$; if $a \neq 0$, then $\| a \| > 0$; (2) $\| 1 + a \| \leqq 1 + \| a \|$; (3) $\| ab \| = \| a \| \cdot \| b \|$; (4) K contains at least one element a for which $\| a \|$ is different from both zero and unity.

The number $\| a \|$ is called the "valuation" (Bewertung) of a. We always find $\| 1 \| = 1 = \| -1 \|$ and $\| a - b \| \geqq \| a - c \| + \| c - b \|$, whence $\| a - b \|$ is a special case of the Mengenlehre concept "Ecart." In order to carry over the concepts of limit and fundamental series we need only substitute Bewertung for absolute value in the usual definitions. If in an "evaluated" realm K every fundamental series has a limit, then K is called perfect. The object of the investigation is to show that by adjunction of new elements every evaluated realm can be enlarged into a perfect one, which, moreover, is algebraically closed. This object is attained by generalizing various investigations of G. Cantor, E. Steinitz, J. Hadamard, and K. Weierstrass.

BATEMAN, H.: "Some equations of mixed differences occurring in the theory of probability and the related expansions in series of Bessel's functions."

A set of objects whose number is constantly increasing are distributed among a large number of boxes, there being no restriction as to the number in each box. The objects are marked with the numbers ± 1 and it is an even chance that a particular object has the mark $+1$. When the average number of objects in a box is a the chance that a box, chosen at random, contains numbers adding up to n is supposed to be a known function $f(n)$. It is required to find the corresponding chance $F_n(x)$ for the case when the average number of objects has increased to x. It is found that when the number of boxes is treated as very large the function F_n satisfies the equation of mixed differences

$$\frac{dF_n}{dx} = \tfrac{1}{2}[F_{n-1} + F_{n+1} - 2F_n].$$

By aid of the particular solution $e^{-(x-a)} I_{n-m}(x-a)$, the general solution is obtained in the form

$$F_n(x) = e^{-(x-a)} \sum_{n=-\infty}^{\infty} I_{n-m}(x-a) f(m).$$

Interesting expansions in series of Bessel's functions $I_m(x)$ are obtained from this result by using particular solutions $F_n(x)$. Problems leading to the difference equations

$$\frac{dF_n}{dx} = F_{n-1} - F_n, \quad \frac{d^2F_n}{dt^2} = k^2(F_{n+1} + F_{n-1} - 2F_n)$$

are treated similarly. The last equation occurs in the writings of John Bernoulli, d'Alembert, and Euler.

PETROVITCH, M.: "Fonctions implicites oscillantes."

An extension of Sturm's theorem on the zeros of the integrals of a linear equation of the second order enables the author to state simple, practical rules capable of putting in evidence the oscillating character and frequency of oscillation of implicit functions, in particular of the integrals which, together with their derivatives, are real, finite, and continuous in a given interval of the independent variable for an infinity of ordinary simultaneous differential equations of whatever order.

Among these equations are some which we meet in problems of dynamics.

HADAMARD, J.: "Sur la série de Stirling."

The problem is to replace a divergent series by a convergent one possessing the same asymptotic properties. Two methods have been employed: (1) transformation into a definite integral (Borel's method); (2) the method of the present paper—adding to each term a correction *which must be inferior in magnitude* (*for* $x = \infty$) *to every term of the given series*, thus securing convergence in a way analogous to Mittag-Leffler's in his theorem on the representation of meromorphic functions by series of rational fractions.

The asymptotic expression of $\log \Gamma(x)$ is closely allied with Raabe's integral $\int_x^{x+1} \log \Gamma(x)$ and by using the results of a memoir of Lindelöf,* the difference between this integral and $\log \Gamma(x)$ was found to be equal to

$$\int_0^\infty \frac{\log (x + it) - \log (x - it)}{2i(e^{2\pi t} - 1)}\, dt.$$

Expanding the numerator in powers of t, we get precisely Stirling's series. To replace that divergent development by a convergent one, we need only separate the integral into two parts

$$\int_0^\infty = \int_0^x + \int_x^\infty.$$

Then

$$\int_0^x \frac{\log (x + it) - \log (x - it)}{2i(e^{2\pi t} - 1)}\, dt =$$

$$\Sigma \frac{(-1)^{n-1}}{2n-1}\left(\frac{B_n}{4n} - \int_x \frac{t^{2n-1}}{e^{2\pi t} - 1} dt\right).$$

Expanding $1/(e^{2\pi t} - 1)$ in powers of $e^{-2\pi t}$ gives the desired result

$$\log \Gamma(x+1) = \log \sqrt{2\pi} - x + (x + \tfrac{1}{2}) \log x$$

$$+ \Sigma(-1)^{n-1}\left(\frac{B_n}{(4n-1)4n} - \varphi_n(x)\right) + R.$$

* *Acta Soc. Fennicæ*, vol. 31 (1912).

where the φ_n are polynomials of degree n in both $1/x$ and $e^{-2\pi x}$, therefore *of an order of magnitude (for $x = \infty$) inferior to that of any power of* $1/x$; the series is convergent; and

$$R = \int_x^\infty \frac{\log(x+it) - \log(x-it) + e^{-\pi t}[\log(x+ite^{-\pi t}) - \log(x-ite^{-\pi t})]}{2i(e^{2\pi t}-1)}\,dt$$

decreases (as x increases) at least as $e^{-2\pi x}$, therefore more rapidly than any power of $1/x$.

SCHLESINGER, L.: "Ueber eine Aufgabe von Hermite aus der Theorie der Modulfunktionen."

FIELDS, J. C.: "Direct derivation of the complementary theorem from elementary properties of rational functions."

FRIZELL, A. B.: "Axioms of ordinal magnitudes."

In this paper the author submits a set of eighty axioms intended to cover all assumptions needed for postulating well ordered types of order. A corollary is that it is not possible to postulate all well ordered types.

PADOA, A.: "Une question de maximum ou de minimum."

Let

$$z = y_1 y_2 \cdots y_n,$$

where

$$y_r = a_{r1}x_1 + a_{r2}x_2 + \cdots + a_{r\,n-1}x_{n-1} + a_{rn};$$

and put

$$n\Delta = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdot & \cdot & \cdots & \cdot \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}.$$

We may always assume $\Delta \geqq 0$.

Denoting by A_{rs} (inclusive of sign, + or − according as $r+s$ is even or odd) the minor of a_{rs}, let us consider only the case in which

$$A = A_{1n}A_{2n}\cdots A_{nn} \neq 0$$

and the variables $x_1, x_2, \cdots, x_{n-1}$ take those values only for which

$$A_{rn}y_r \geqq 0.$$

By purely elementary considerations it is shown that z always

reaches either a maximum or a minimum according as A is positive or negative, that in both cases this value of z is Δ^n / A and corresponds to the values

$$x_s' = \frac{1}{n}\left(\frac{A_{1s}}{A_{1n}} + \frac{A_{2s}}{A_{2n}} + \cdots + \frac{A_{ns}}{A_{nn}}\right)$$

of $x_1, x_2, \cdots, x_{n-1}$.

The theorem is applied to determine the interior point of a tetraedron the product of whose distances from the faces is a maximum.

STERNECK, R. VON: "Neue empirische Daten über die zahlentheoretische Funktion $\sigma(n)$."

F. Mertens established the relation $|\sigma(n)| \leqq \sqrt{n}$ for all values of n up to 10,000 and showed that a general proof of it would justify Riemann's conjecture that the zeros of $\zeta(s)$ all have the real part $\frac{1}{2}$.

The author showed (1897–1901) by tabulating $\sigma(n)$ up to $n = 500{,}000$ that, except for a few values in the neighborhood of $n = 200$, the quotient $\sigma(n)/\sqrt{n}$ oscillates in this interval between ± 0.46.

More recently, with the support of the Imperial Academy at Vienna, he has had 16 new values of σ computed, ranging from $n = 500{,}000$ to $n = 5{,}000{,}000$. This computation gives for $|\sigma(n)|/\sqrt{n}$ the values: 0.297, 0.270, 0.022, 0.237, 0.214, 0.140, 0.209, 0.133, 0.302, 0.175, 0.230, 0.063, 0.073, 0.097, 0.083, 0.315, thus increasing the probability that Riemann's guess will prove correct and furnishing reason to expect that the limits of oscillation for $\sigma(n)/\sqrt{n}$ will turn out to be the same up to $n = 5{,}000{,}000$ as in the smaller interval previously examined.

ELLIOTT, E. B.: "Some uses in the theory of forms of the fundamental partial fractions identity."

If $F(\varphi) \equiv (\varphi - a_1)(\varphi - a_2)\cdots(\varphi - a_n)$, where φ is a symbol of direct differential operation, $\varphi(x, y, z, \cdots, \partial/\partial x, \partial/\partial y, \partial/\partial z, \cdots)$, and $a_1, a_2, \cdots, a_n$ constants, and if u is a solution of the differential equation

$$F(\varphi)u = f(x, y, z, \cdots), \tag{1}$$

the identity

$$u = \sum_{s=1}^{s=n}\left\{\frac{1}{F'(a_s)} \cdot \frac{F(\varphi)}{\varphi - a_s} u\right\}$$

expresses u as a sum $u_1 + u_2 + \cdots + u_n$, where, for $s = 1, 2, \cdots, n$, u_s satisfies

$$(\varphi - a_s)u_s = \frac{1}{F'(a_s)} f(x, y, z, \cdots)$$

in virtue of (1). The parts u_s are obtained by *direct operation*. This fact has numerous applications in the theory of forms. Three of them are exhibited in this paper. In all the right-hand side of (1) is zero.

I. The general rational integral homogeneous isobaric function of degree i and weight w in the coefficients of $(a_0, a_1, \cdots, a_p)(x, y)^p$ is separated by direct operation into $w + 1$ parts.

II. The most general rational integral function of $a_1, a_2, \cdots, a_p$ of degree i is separated into $ip + 1$ parts, which are orthogonal invariants with the various possible multiplying factors $e^{\iota(ip-2m)}$, $\iota = \sqrt{-1}$, $m = 0, 1, 2, \cdots, ip$, in the expression of invariancy for the direct transformation

$$x = X \cos \vartheta - Y \sin \vartheta, \quad y = X \sin \vartheta + Y \cos \vartheta,$$

III. The most general rational integral function of degree i and equal first and second weights q, q in the double system of coefficients

$$\begin{matrix} c_{00} \\ c_{10},\ c_{01} \\ c_{20},\ c_{11},\ c_{02} \\ \cdot\ \cdot\ \cdot\ \cdot \end{matrix}$$

is directly operated on in such a way as to produce from it the most general gradient of the type i, q, q which has two, Ω_{xz}, Ω_{yz}, of the four annihilators of covariants of ternary quantics

$$\sum_{r=0,\, s=0}^{r+s=p} \frac{p!}{r!\, s!(p-r-s)!} c_{rs} x^r y^s z^{p-r-s},$$

p being any number sufficiently large to allow all the coefficients c_{rs} in the gradient to be present in the quantic.

KOCH, H. VON: "On regular and singular solutions of certain infinite systems of linear equations."

WHITTAKER, E. T.: "On the functions associated with the elliptic cylinder in harmonic analysis."

This paper is devoted to the "elliptic cylinder functions" defined by the equation

$$\frac{d^2y}{dz^2} + (a + k^2 \cos^2 z)y = 0,$$

where a and k denote constants. This equation presents itself in the theory of linear differential equations as the most natural one to discuss after the hypergeometric equation. Its periodic solutions are shown to be the same as the solutions of the homogeneous integral equation

$$y(z) + \lambda \int_0^{2\pi} e^{k \cos z \cos s}\, y(s)ds = 0.$$

Making this equation the basis of the further developments, the author obtains directly from it the expressions of the elliptic cylinder functions. The one of lowest order (which degenerates into the Bessel's function of zero order when the elliptic cylinder degenerates into a circular cylinder) is

$$ce_0(z) = 1 + \left(\frac{1}{8}k^2 - \frac{7}{2^{13}}k^6 + \cdots\right)\cos 2z + \cdots$$
$$+ \left(\frac{k^{2r}}{2^{4r-1}r!\,r!} - \frac{r(3r+4)k^{2r+4}}{2^{4r+7}(r+1)!\,(r+1)!} + \cdots\right)\cos 2rz.$$

SALTYKOW, N.: "Sur l'intégration des équations aux dérivées partielles."

Let there be a normal system of q equations

(1) $f_i(x_1, x_2, \cdots, x_n, p_1, p_2, \cdots, p_n) = 0 \quad (i = 1, 2, \cdots, q)$

solvable with respect to $p_1, p_2, \cdots, p_q$, the corresponding linear system

(2) $$(f_i, f) = 0 \qquad (i = 1, 2, \cdots, q),$$

admitting $n + \rho (\rho < n - q)$ distinct integrals

(3) $$f_1, f_2, \cdots, f_q, f_{q+1}, \cdots, f_{n+\rho}.$$

S. Lie has integrated the system (1) by a quadrature when the integrals (3) satisfy a certain condition, which may be replaced by

(4) $$dz = \sum_{s=1}^{n} p_s dx_s,$$

provided this expression becomes an exact differential in virtue of equations (1) *and the* $n - q + \rho$ *equations obtained by setting the last* $n - q + \rho$ *integrals* (3) *equal to arbitrary constants*

$a_1, a_2, \cdots, a_{n-q+\rho}$. The question then arises whether this restriction diminishes the generality of the solution. It is shown that the integrals obtained from the system (2) are the same on Lie's hypothesis as on that of the author. The latter is sufficient to yield the complete integral of (1) without making use of the complete system of integrals of (2). Moreover it becomes possible to dispense with the functional group. For if we have

$$(5) \qquad z = \varphi(x_1, x_2, \cdots, x_{n-\rho}, a_1, a_2, \cdots, a_{n-q+\rho}) + a$$

$$(6) \quad x_{n-\rho+i} = \varphi_i(x_1, x_2, \cdots, x_{n-\rho}, a_1, a_2, \cdots, a_{n-q+\rho}) \quad (i = 1, 2, \cdots, \rho)$$

and

$$(7) \quad p_s = \psi_s(x_1, x_2, \cdots, x_{n-\rho}, a_1, a_2, \cdots, a_{n-q+\rho}) \quad (s = 1, 2, \cdots, n),$$

it follows that

$$\frac{\partial \varphi}{dx_r} = \psi_r + \sum_{i=1}^{\rho} \psi_{n-\rho+i} \frac{\partial \varphi_i}{\partial x_r}, \quad (r = 1, 2, \cdots, n - \rho).$$

The necessary and sufficient conditions that eliminating the constants $a_{n-q+1}, a_{n-q+2}, \cdots, a_{n-q+\rho}$ *from equations* (5) *and* (6) *furnish the complete integral of* (1) *are given by*

$$(8) \qquad D\left(\frac{\varphi_1, \varphi_2, \cdots \varphi_\rho}{a_{n-q+1}, a_{n-q+2}, \cdots a_{n-q+\rho}}\right) \gtrless 0$$

$$U'_{a_{n-q+i}} \equiv 0 \quad (i = 1, 2, \cdots, \rho), \quad U'_a \gtrless 0, \quad U'_{a_k} \gtrless 0 \quad (k = 1, 2, \cdots, n - q),$$

where

$$(9) \qquad U'_{a_s} \equiv \frac{\partial \varphi}{\partial a_s} - \sum_{i=1}^{\rho} \psi_{n-\rho+i} \frac{\partial \varphi_i}{\partial a_s}.$$

When these last conditions are satisfied the first n integrals (3) *are in involution.*

Whatever the integrals (3) *may be, it is always possible to satisfy the conditions* (8) *by introducing in formulas* (5)–(7) *new arbitrary constants to denote the initial values of the variables.*

If we have to do with any normal system whatever of partial equations it is unnecessary to seek the complete integration of the corresponding linear system, whenever the integrals obtained satisfy the conditions of the above theorem, since the other integrals may then be obtained either by means of formulas (9) or by aid of Jacobi's theorem generalized.

These results were applied to Imschenetsky's equation

$$F \equiv (x_2p_1 + x_1p_2)x_3 + dp_3(p_1 - p_2) = a,$$

showing how to complete the integration by algebraic elimination.

RÉMOUNDOS, G.: "Sur les singularités des équations différentielles."

When an integral is sought which is to vanish for $x = 0$ we sometimes meet with an interesting singularity characterized by the following properties:

1°. It is possible to obtain from the differential equation a power series that satisfies it formally.

2°. This series is, in general, divergent and consequently there is, in general, no integral which is holomorphic in the neighborhood of $x = 0$ and vanishes at this point. These two properties are consequences of the following, which suffices to characterize the singularity in question: The terms of the differential equation which predominate in computing coefficients of the series by successive differentiation do not have the maximum order as it occurs in the regular cases or the ordinary singularities. Introducing the notion of weight (poids) the characteristic property may be expressed as follows: *No term of maximum order has the maximum weight.* Rules are given in the paper for determining the weights of various terms in the differential equation. This singularity presents itself in equations of the form

$$y = bx + \alpha x^2y' + F[x, y, y', y'', \cdots, y^{(m)}],$$

where F denotes a function developable in power series in $x, y, y', y'', \cdots, y^{(m)}$ of which the weight is *negative.* If the coefficients α, b, and those of F are all *positive,* there exists no integral of this equation that vanishes at $x = 0$ and is holomorphic in its neighborhood.

HILL, M. J. M.: "The continuation of the hypergeometric series."

The object of this paper is to call attention to certain difficulties which arise in the attempt to apply the *method of ordinary algebraic expansion,* which has been successfully applied to series having one or two points of singularity, to a series with three such points, viz.: the hypergeometric series.

The equation to be proved is

$$F(\alpha, \beta, \alpha+\beta-\gamma+1, 1-x)=\frac{\pi(\alpha+\beta-\gamma)\pi(-\gamma)}{\pi(\alpha-\gamma)\pi(\beta-\gamma)}F(\alpha, \beta, \gamma, x)$$

$$+\frac{\pi(\gamma-2)\pi(\alpha+\beta-\gamma)}{\pi(\alpha-1)\pi(\beta-1)}x^{1-\gamma}\cdot F(\alpha-\gamma+1, \beta-\gamma+1, 2-\gamma, x).$$

The series $F(\alpha, \beta, \alpha+\beta-\gamma+1, 1-x)$ and $x^{1-\gamma}\cdot F(\alpha-\gamma+1, \beta-\gamma+1, 2-\gamma, x)$ cannot be expanded in series of integral powers of x. Consequently in each series concerned only $n+1$ terms are taken, and an attempt is made to prove that the difference between the two sides tends to zero as n tends to ∞, it being known that $|x|$ and $|1-x|$ are each less than unity. We expand $x^{1-\gamma}$ in powers of $1-x$ and take $n+1$ terms. Then, multiplying by $F(\alpha-\gamma+1, \beta-\gamma+1, 2-\gamma, x)$ it is shown that the terms of degree higher than n can be neglected. The right hand side is thus reduced to a polynomial of degree n in x, and $F(\alpha, \beta, \alpha+\beta-\gamma+1, 1-x)$ is treated in the same way.

The coefficient of x^r is then transformed into two parts, one of which is the coefficient of x^r in

$$\frac{\pi(\alpha+\beta-\gamma)\pi(-\gamma)}{\pi(\alpha-\gamma)\pi(\beta-\gamma)}F(\alpha, \beta, \gamma, x).$$

The difference between the other part of the coefficient of x^r in $F(\alpha, \beta, \alpha+\beta-\gamma+1, 1-x)$ and the coefficient of x^r in

$$\frac{\pi(\gamma-2)\pi(\alpha+\beta-\gamma)}{\pi(\alpha-1)\pi(\beta-1)}x^{1-\gamma}F(\alpha-\gamma+1, \beta-\gamma+1, 2-\gamma, x)$$

is expressed in the form $k_0(1-x)^n + k_1x(1-x)^{n-1} + k_2x^2(1-x)^{n-2} + \cdots + k_nx^n$ and it is clear that when v is small compared with n the term $k_vx^v(1-x)^{n-v}$ tends to zero as n tends to infinity. But when v is comparable with n further investigation is needed.

CUNNINGHAM, A.: "On Mersenne's numbers."

A Mersenne's number is $M_q = 2^q - 1$ when q is prime. Taking the 56 primes $q = 1, 2, 3, \cdots, 257$, Père Mersenne affirmed (in 1644) that only 12 values of q made M_q prime, viz., $q = 1, 2, 3, 5, 7, 13, 17, 19, 31, 67, 127, 257$; and that the remaining $44 M_q$ were all composite. The grounds for this

assertion are not known, and it has up to the present time been only partially verified. Twelve of the M_q have been proved prime, viz., those for which $q = 1, 2, 3, 5, 7, 13, 17, 19, 31, 61$ (not 67), 89 and 127. Twenty-nine have been proved composite and all factors found for eleven of them, viz., those given by $q = 11, 23, 29, 37, 41, 43, 47, 53, 59, 67, 71$; while one or more factors have been found for the eighteen given by $q = 73, 79, 83, 97, 113, 131, 151, 163, 173, 179, 181, 191, 197, 211, 223, 233, 239, 251$. The first factors when $q = 71, 163, 173, 197$ were found by the author.

EVANS, G. C.: "Some general types of functional equations."

BECKH-WIDMANSTETTER, H. A. VON: "Eine neue Randwertaufgabe für das logarithmische Potential."

Hitherto only those boundary value problems have been considered where the function itself or a derivative in a fixed direction is given on the boundary. The author discusses in this paper a boundary condition

$$Ax^2\frac{\partial^2 z}{\partial x^2} + 2Bxy\frac{\partial^2 z}{\partial x \partial y} + Cy^2\frac{\partial^2 z}{\partial y^2} = F(x, y).$$

Applying Fourier's series

$$z = b_0 + \sum_{k=1}^{\infty} r^k(a_k \sin k\varphi + b_k \cos k\varphi)$$

$$F(x, y) = f(\varphi) = d_0 + \sum_{k=1}^{\infty}(c_k \sin k\varphi + d_k \cos k\varphi),$$

the solution is undetermined as to a solution U of

$$Ax^2\frac{\partial^2 U}{\partial x^2} + 2Bxy\frac{\partial^2 U}{\partial x \partial y} + Cy^2\frac{\partial^2 U}{\partial y^2} = 0.$$

By aid of elementary transcendentals U can be expressed linearly in 8 arbitrary constants. Instead of the method of varying parameters, the author has given a direct treatment of the difference equation with constant coefficients, which yields more convenient expressions. He obtains the Fourier series for z and sums it by introducing

$$c_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(\psi)\sin k\psi d\psi, \quad \text{etc.}$$

Thus the solution is carried up to the expression as definite integral after the analogy of Poisson's. The author refers, for these complicated expressions, to his own work in the *Monatshefte für Mathematik und Physik*, volume 23 (1912).

PEDDIE, W.: "A mechanism for the solution of an equation of the nth degree."

The principle employed may be seen from a consideration of that system of pulleys in which each of several cords has one end fastened to the body to be raised while its other end carries one of the pulleys of the system. If the body be moved up through a distance a the free end of the last cord descends by the amount $a(2^n - 1)$. Suppose now that the body is immovable while the cords are wound on drums attached to it. If a length a be unwound from the first drum, the free end of the last cord descends through the distance $2^n a$. If a length b be unwound from the second drum, the free end descends by an additional amount $2^{n-1}b$, and so on. Thus, if the length of cord let off the last drum be such that the free end of the last cord has retaken its initial position, we may say that the arrangement solves the equation

$$ax^n + bx^{n-1} = 0$$

in the particular case in which $x = 2$.

If the direction of motion of the drums and the direction of motion of the pulleys are inclined to each other at an angle $\pm\varphi$ when the free end of the last cord has retaken its initial position the corresponding root of the equation is $1 \pm \sin \varphi$.

The cords are replaced by fine steel wires and the action of gravity by the control of wires wound on spring drums. The free end of the last cord is also wound on a spring drum provided with a scale and index. The instrument thus readily gives the value of the function for any value of the variable and conversely.

VOLTERRA, V.: "Sopra equazioni di tipo integrale."

In the absence of Professor Volterra part of his paper was read by Professor Somigliana.

WILKINSON, M. M. U.: "Elliptic and allied functions; suggestions for reform in notation and didactical method."

The paper suggests that a notation based on Weierstrass's

sigma formulæ should supersede the sn., cn., dn. notation, which superseded the sinam, cosam, Δam notation many years ago. It also suggests that the σ-function should be defined by the differential equation

$$\sigma\frac{d^4\sigma}{du^4}-4\frac{d\sigma}{du}\frac{d^3\sigma}{du^3}+3\left(\frac{d^2\sigma}{du^2}\right)^2+g_1\left\{\sigma\frac{d^4\sigma}{du^4}-\left(\frac{d\sigma}{du}\right)^2\right\}-\tfrac{1}{2}g_2\sigma^2=0.$$

It concluded with the suggestion that more attention than has hitherto been customary should be paid to the differential equation

$$\tau\frac{d^4\tau}{du^4}-4\frac{d\tau}{du}\frac{d^3\tau}{du^3}+3\left(\frac{d^2\tau}{du^2}\right)^2=0.$$

Throughout the paper was illustrated by examples of formulæ and investigations simplified by the proposed reforms.

ZERVOS, P.: "Sur les équations aux dérivées partielles du premier ordre à quatre variables."

The purpose of this communication was to prove three propositions that are important because of their relation to the calculus of variations.

1°. To every partial differential equation of the first order in three independent variables

$$F(x_1, x_2, x_3, x_4, p_1, p_2, p_3) = 0 \tag{1}$$

corresponds a Monge's equation of the second order

$$f_1(x_1, x_2, x_3, x_4, dx_1, dx_2, dx_3, dx_4, d^2x_1, d^2x_2, d^2x_3, d^2x_4,) = 0. \tag{2}$$

2°. The general solution of equation (2) can be found immediately if we know the complete integral of (1)

$$V(x_1, x_2, x_3, x_4, a_1, a_2, a_3) = 0. \tag{3}$$

3°. In two communications presented to the Paris Académie des Sciences, April 10 and September 23, 1905, the author gave some general results on Monge equations of the first order from which it follows that to the equation (1) we may assign a Monge's equation of the first order

$$f(x_1, x_2, x_3, x_4, dx_1\, dx_2, dx_3, dx_4) = 0, \tag{4}$$

equivalent to the equations

$$V=0,\quad \frac{\Delta V}{\Delta\alpha}=0,\quad \frac{\Delta^2 V}{\Delta\alpha^2}=0,\quad \sum_{\kappa=1}^{4}\frac{\partial}{\partial x_\kappa}\left(\frac{\partial V/\partial a_1}{\partial V/\partial a_3}\right)dx_\kappa=0.$$

Consequently the equations (2) and (4) have in their solutions three equations in common, namely

$$V = 0, \quad \frac{\Delta V}{\Delta \alpha} = 0, \quad \frac{\Delta^2 V}{\Delta \alpha^2} = 0.$$

From this there follow some important geometrical theorems.

RABINOVITCH, G.: "Eindeutigkeit der Zerlegung in Primzahlfaktoren in quadratischen Zahlkörpern."

The paper treats quadratic realms having $D = 1 - 4m$.

Theorem 1.—Given, in a certain realm, two integers α and β of which neither is divisible by the other, if it is possible to find two other integers ξ and η of the same realm satisfying the inequality

$$0 < N(\alpha\xi - \beta\eta) < N\beta,$$

then the class number of the realm is unity.

Definition.—A fraction α/β will be called "disturbing" (störend) when the inequality

$$0 < N\left(\frac{\alpha}{\beta}\xi - \eta\right) < 1$$

admits no solutions in integers.

Theorem 2.—If the class number of a realm is greater than unity, this realm contains disturbing fractions of the form $(p - \vartheta)/q$ where $p < q < m$.

Theorem 3.—The class number of a realm is greater than or equal to one according as the sequence of $m - 1$ numbers $p^2 - p + m$ $(p = 1, 2, \cdots, m - 1)$ does or does not contain composite numbers.

PEEK, J. H.: "On an elementary method of deducing the characteristics of a certain partial differential equation of the second order."

MARTIN, A.: "On powers of numbers whose sum is the same power of some number."

ARTHUR B. FRIZELL.

SHORTER NOTICES.

Elliptische Funktionen. Von Professor Dr. KARL BOEHM. Zweiter Teil. Göschen (Sammlung Schubert LXI), Leipzig, 1910. vii+180 pp. M. 5.

THIS little book is complete in itself and independent of the first volume.* It is devoted entirely to the older theory, that is, the development of the properties of elliptic integrals and the inversion problem.

In the first chapter we have a rather thorough discussion of hyperelliptic integrals. There is, first, the reduction of the general hyperelliptic integral to the three possible types and, second, the classification of these integrals according to deficiency (Geschlecht). The elliptic integrals are thus introduced as a particular class of hyperelliptic integrals (cf. Picard, Traité, Tome I).

The second chapter deals with the behavior of the elliptic integral of the first kind in the complex plane and upon the proper Riemann surface, leading to the existence and meaning of the two period moduli.

The third chapter is devoted to the inversion of the elliptic integral of the first kind. The point of departure is the theorem concerning the existence of the solution of the differential equation $dz/du = F(z)$, where $F(z)$ is an analytic function of z.

A careful proof of the uniqueness of the inverse function is given; the final conclusion being that this function is meromorphic in the entire u plane. The results of the preceding chapter enable the author to add that the inverse function is doubly periodic and of the second order. In this way, then, we are led to the elliptic function of the second order. The chapter closes with the theorem:

The most general elliptic integral can be expressed as the integral of an elliptic function $\phi(u)$, where u is the elliptic integral of first kind belonging to the same irrationality as appears in the general integral.

The fourth chapter considers the integral of the first kind and its inversion when the branch points are real. There is

*Reviewed in the BULLETIN for January, 1911, vol. 17, p. 202.

a discussion of the real values of the inverse function and the various ways of representing this function geometrically.

Chapter five is devoted to a discussion of the usual normal forms of the elliptic integral of the first kind.

The sixth and last chapter contains Abel's theorem and its consequences; in particular addition theorems for the inverse function and for related functions.

As is stated in the preface, the book contains no discussion of modular functions, the transformation theory, or the applications. The author's purpose is to present, in a concise manner, the necessary concepts and theorems which form the definition and fundamental properties of elliptic integrals and their inverses. This he has succeeded in doing without digression, although the temptation to include one or more chapters on the subjects noted above must have been great.

The beginner will better appreciate the first volume if he follows Dr. Boehm's suggestion (Vorwort, Erster Teil, page iv), viz., to read this second volume first.

L. WAYLAND DOWLING.

Éloges académiques et Discours. Volume publié par le Comité du Jubilé scientifique de M. GASTON DARBOUX. Paris, A. Hermann et Fils, 1912. 525 pp. Fr. 5.

THE present volume was published by an international committee of mathematicians, formed for the purpose of expressing an appreciation of the scientific work of M. Gaston Darboux at the completion of his fiftieth year of public instruction. This committee addressed a circular letter to the mathematicians of all countries, inviting subscriptions for the purpose of awarding M. Darboux a medal on this occasion. The responses were so numerous and so liberal that the committee was enabled to have the medal executed by the eminent French artist M. Vernon, and also to publish the present volume and send it to all the subscribers.

The greater part of the volume consists of a collection of eulogies by M. Darboux. The subjects of these are: Joseph-Louis-François Bertrand, François Perrier, Charles Hermite, Antoine d'Abbadie, Général Meusnier, Donateurs de l'Académie. These eulogies are followed by a collection of discourses by M. Darboux and by an account of the Jubilé, including the various addresses and a list of the subscribers. The Jubilé was to be held towards the end of October, 1911,

but it was postponed until January 21, 1912, on account of the death of Mme. Darboux on October 8, 1911.

The addresses which were delivered at the Jubilé give abundant evidence of the great influence of M. Darboux as a teacher. Several of them direct attention to his fundamental contributions to the advancement of mathematical knowledge. The address by M. Henri Poincaré is especially interesting in this direction. The American representatives on the committee in charge of this Jubilé were G. E. Hale and H. Hancock.

G. A. MILLER.

Applications of the Calculus to Mechanics. By E. R. HEDRICK and O. D. KELLOGG. Ginn and Company, 1910. 116 pp.

IN the mathematical courses given to engineering students the analytic procedures find applications to geometry, and here the applications often end. Hedrick and Kellogg through their book, Applications of the Calculus to Mechanics, have given material help toward eliminating this mistake.

The book is clearly written, and for the most part in such a way that the student after his first course in the calculus will be able to read it understandingly. It is refreshing to find an accurate treatment of subjects in mechanics in which the authors have evidently kept their prospective readers in mind while writing it. It shows that after all there is no inherent reason why such a treatment may not be accurate, and at the same time clear to the student who is to read it. With the exception of a few paragraphs the authors seem to me to have put into their book these two essential qualities of a good text book. There are a few paragraphs which, I think, are too condensed, in which too much is left to the student. To illustrate what is meant, on page seven it is stated that "If a vector vary with the time t, or any other parameter, its derivative may be defined, for we know how to subtract vectors and divide by numbers. The notion of limit of a set of vectors will be sufficiently clear." The rest is left to the student. However, even if my supposition here is correct, this is only one of a few isolated cases and the teacher can easily supply the necessary amplifications. In nearly the whole of the book the definitions and theorems are led up to in such a plausible way, and so well illustrated by examples worked out and fully explained in the text, that

the student is in a position to comprehend the subjects treated and to use the theory developed in the formulation and solution of problems for himself.

The book contains five chapters: Vectors; Statics; Dynamics of a particle; Work and energy; Mechanics of rigid bodies. Center of mass is treated in Chapter II, and moments of inertia in Chapter V. There are fifteen sets of problems under the different subjects treated. These sets contain in all two or three hundred carefully selected and well graded problems.

I have been greatly aided by this book during the past two years in teaching the calculus to engineering students.

DAVID C. GILLESPIE.

NOTES.

THE closing (October) number of volume 13 of the *Transactions of the American Mathematical Society* contains the following papers: "On the pseudo-resolvent to the kernel of an integral equation," by W. A. HURWITZ; "Infinite systems of indivisible groups," by G. A. MILLER; "Improper multiple integrals over iterable fields," by J. K. LAMOND; "On a theorem of Fejér and an analogon to Gibbs' phenomenon," by T. H. GRONWALL; "The southerly and easterly deviations of falling bodies for an unsymmetrical gravitational field of force," by W. H. ROEVER; "On approximation by trigonometric sums and polynomials," by DUNHAM JACKSON. Also notes and errata, volumes 7 and 13.

THE annual meeting of the American association for the advancement of science will be held in Cleveland during the week beginning December 30 under the presidency of Professor E. C. PICKERING, of Harvard University; Professor E. B. VAN VLECK, of the University of Wisconsin, is chairman of Section A. It is expected that a large number of societies will affiliate with the association for this meeting.

THE annual meeting of the French association for the advancement of science was held at Nîmes, August 1–7, under the presidency of Professor CH. GARIEL. The section in mathematics and allied subjects was presided over by Pro-

fessor E. LEBON. In addition to several papers of non-mathematical content, the following were presented: M. LITRE, "Theory of the Foucault pendulum"; A. AUBRY, three papers on "The errors of mathematicians," and "The principles of the theory of complex numbers"; A. GÉRARDIN, "On methods of solution employed in the theory of numbers," and "On a new algebraic machine"; G. TARRY, "Tables of triple entry of divisors of numbers from 1 to N"; A. PELLET, "On partial differential equations," and "On asymptotic lines"; C. A. LAISANT, "On the tables of divisors"; L. AUBRY, "Direct demonstration that every prime number of the form $4n + 1$ is the sum of two squares," "Method of solving $x^2 - ay^2 = 1$," and "A method for the decomposition of numbers"; L. FAVRE, "On the errors of mathematicians"; L. TRIPIER, "An application of successive approximation to the solution of numerical equations"; E. N. BARISIEN, "On certain summations and series"; A. CHRÉTIEN, "Tables of polynomials of Legendre"; WELSCH, "Diametral lines of algebraic curves," "The circles of Joachimsthal," "On the theorem of Fuss," and "Steiner polygons inscribed in a quartic, and their relation to those of Poncelet"; F. BOULAD, "On the equations in 4 variables of nomographic order 4."

The next meeting of the association will be held in Tunis, March 22–27, 1913. Professor A. GAUTHIER was elected vice-president of the association and M. MAINGROT, of Tunis, president of the mathematical section.

THE Italian society for the advancement of science held its annual meeting at Genoa during the week October 17–24. The programme included the following papers of mathematical nature:

General lectures: "A new theory of universal attraction," by M. ABRAHAM; "Is the history of sciences a science," by G. LORIA; "The third law of thermodynamics," by L. ROLLA. Section I (mathematics, physics, mineralogy): "A new type of integro-differential equations," by L. AMOROSO; "A theorem of reciprocity for some functions analogous to Green's function in the theory of elasticity," by T. BOGGIO; "On improper definite integrals," by E. BORTOLOTTI; "Plane quintics invariant under a group of collineations," by E. CIANI; "Recent researches in hydrodynamics," by U. CISOTTI. Section XV (history of the sciences): "Correspondence of

Paolo Ruffini," by E. BORTOLOTTI; "An unpublished translation of the writings of Archimedes among the Galilean manuscripts in the national library of Florence," by A. FAVARO; "On semiregular polyhedra," by G. LORIA; "Synthetic theory of real numbers in a text of G. A. Borelli (17th century)," by F. PODESTI; "Archimedes in China," by G. VACCA.

The two sections unanimously adopted resolutions proposed by Professors Loria and Volterra, urging that in the complete edition of Euler's works, now in publication, there should be included the comments of Lorenzo Mascheroni on the integral calculus, as has been done with Lagrange's additions to the work on algebra, and that the Italian government be requested to grant a subvention, if necessary, to meet the cost of this enlargement of the original plan of publication.

Meetings of other scientific societies were also held in Genoa at the same time, among them the Italian mathematical society "Mathesis," at whose meeting the following lectures were delivered: "The school in its connection with life and modern science," presidential address by G. CASTELNUOVO; "Eccentricities and mysteries of numbers," by G. LORIA; "Mathematical precision and approximation," by V. REINA; "The classical authors of mathematics," by G. VACCA.

Reports of the Italian subcommittee of the International commission on the teaching of mathematics were also presented and discussed: Primary school, by A. CONTI; classical teaching, by A. FAZZARI and U. SCARPIS; technical teaching, by G. SCORZA; commercial and trade schools, by G. LAZZERI; preparation of teachers, by S. PINCHERLE.

The society recorded its views on various desirable reforms in the teaching of mathematics in Italy. A joint meeting was held with the Italian association of electrical engineers and the Italian physical society for discussion of the organization of mathematical instruction for engineering students. A committee was appointed to prepare concrete recommendations. A joint meeting with the Italian philosophical society was devoted to a discussion of the concept of infinity.

THE following university courses in mathematics are announced:

CAMBRIDGE UNIVERSITY.—By Professor E. W. HOBSON: Spherical harmonics and allied functions; Integral equations;

The history of the problem of the "squaring of the circle" and of related questions.—By Professor G. H. DARWIN: Gravitation with astronomical applications; Lunar theory.—By Professor R. S. BALL: Celestial mechanics; Spherical astronomy.—By Professor J. LARMOR: Electricity and magnetism; Electrodynamics and optical theory.—By Professor J. J. THOMSON: Electricity and matter; Electricity and magnetism; Discharge of electricity through gases.—By Professor B. HOPKINSON: Applied mechanics.—By Professor H. F. NEWALL: Solar research.—By Dr. H. F. BAKER: Introduction to the theory of functions; Geometry of birational transformation; Theory of functions.—By Mr. R. A. HERMAN: Hydrodynamics; Differential geometry; Hydromechanics (*A*).—By Mr. R. W. RICHMOND: Algebraic geometry; Higher solid geometry; Synthetic geometry.—By Dr. T. J. I'A. BROMWICH: Electric waves and electro-optics; Dynamics (*A*); Optics with experimental illustrations; Geometrical and physical optics (*A*); Potential theory and problems.—By Mr. J. H. GRACE: Theory of numbers; Theory of invariants; Elements of Fourier analysis and calculus of variations (*A*).—By Dr. E. W. BARNES: Linear differential equations (*B*).—By Mr. A. J. WALLIS: Spherical trigonometry and astronomy (*A*).—By Mr. A. BERRY: Theory of ordinary differential equations (*B*); Elliptic functions and elementary harmonic analysis (*A*); Elliptic functions (*B*); Theory of transformation of elliptic functions.—By Mr. G. T. BENNETT: Line geometry.—By Mr. A. MUNRO: Hydrodynamics and sound (*A*).—By Mr. B. RUSSELL: The fundamental concepts of mathematics; Principles of mathematics.—By Mr. G. T. LEATHAM: Electron theory.—By Mr. G. H. HARDY: General theory of Dirichlet's series; Asymptotic relations in the theory of functions; Double limit problems.—By Mr. G. BIRTWISTLE: Hydrodynamics (*A*); Hydrodynamics (*B*); Thermodynamics (*B*).—By Mr. F. J. M. STRATTON: Orbits from observations; Stellar physics.—By Mr. J. W. NICHOLSON: Physical optics; Electric waves and theory of diffraction.

OXFORD UNIVERSITY.—By Professor W. ESSON: Analytic geometry of plane curves; Synthetic geometry of plane curves.—By Professor E. B. ELLIOTT: Theory of numbers; Sequences and series.—By Professor A. E. H. LOVE: Electricity and magnetism.—By Professor H. H. TURNER: Elementary

mathematical astronomy.—By Mr. T. W. CHAUNDRY: Solid geometry.—By Mr. A. L. DIXON: Calculus of finite differences; Calculus of variations.—By Mr. J. E. CAMPBELL: Differential equations.—By Mr. A. E. JOLLIFFE: Doubly periodic functions.—By Mr. F. B. PIDDUCK: Analytic statics and attraction.—By Mr. C. H. THOMPSON: Dynamics of particles and rigid bodies.—By Mr. H. T. GERRANS: Hydrodynamics.—By Mr. A. L. PEDDER: Problems in pure mathematics.—By Mr. C. E. HASELFOOT: Theory of equations.—By Mr. C. H. SAMPSON: Plane analytic geometry.—By Mr. J. W. RUSSELL: Differential calculus.—By Mr. E. H. HAYES: Statics and hydrostatics.

THE city of Nancy, in the province of Lorraine, will erect a monument to the memory of J. V. PONCELET, CH. HERMITE, and J. H. POINCARÉ, all of whom were born in this province.

THE Paris academy of sciences has awarded its Binoux prize (for the history of science) to Professor J. L. HEIBERG, of the University of Copenhagen, for his works on the history of ancient mathematics and in particular for those on the Method of Archimedes.

"THE general ideas of the science of geometry" is the title of a non-technical course of lectures being delivered by Dr. A. N. WHITEHEAD during the first and second terms of the present academic year at University College, University of London.

PROFESSOR H. A. LORENTZ, of the University of Leyden, has been chosen an honorary member of the Vienna academy of sciences.

PROFESSOR CARL RUNGE, of the University of Göttingen, has received the distinction of Geheimerregierungsrat.

DR. R. PFEIFFER, of the University of Halle, has been promoted to an associate professorship of mathematics.

PROFESSOR G. HUMBERT, of the Ecole polytechnique, has been made head professor of mathematics in the College of France.

IN the faculty of sciences of the University of Paris the following changes have been announced: Professor E. CARTAN,

formerly in charge of conferences, has been made professor of differential and integral calculus (Ecole normale); Professor E. VESSIOT has been placed in charge of conferences; Professor CL. GUICHARD is in charge of the courses of Professor P. PAINLEVÉ.

THE Italian society of sciences (the forty) has awarded to Professor E. E. LEVI, of the University of Genoa, its gold medal for the most important papers by an Italian mathematician during the period 1907–1911. Professor Levi has also been promoted to a full professorship of the calculus.

PROFESSOR T. BOGGIO, of the University of Turin, has been promoted to a full professorship of theoretical mechanics.

PROFESSOR C. SOMIGLIANA, of the University of Turin, has been elected a member of the Italian society of sciences (the forty).

DR. J. W. NICHOLSON, mathematical lecturer in Girton College, Cambridge University, has been appointed professor of mathematics in King's College, University of London.

MR. H. HILTON has been promoted to a professorship of mathematics in the Bedford College for Women, University of London.

PROFESSOR G. H. ALBRIGHT, professor of mathematics at Colorado College, will be an exchange professor to Harvard University during a portion of the present academic year.

AT the University of Minnesota, Mr. H. B. ROE has been promoted to an assistant professorship of mathematics.

DR. J. E. HODGSON, of the Baltimore Polytechnic Institute, has been appointed associate professor of mathematics in West Virginia University.

AT the University of California Mr. J. E. MADDRILL has been appointed instructor in mathematics.

AT Princeton University Messrs. J. A. ALEXANDER, R. E. GILMAN, and E. S. SMITH have been appointed instructors in mathematics.

MR. H. M. DOUGLASS, of Cornell University, has been appointed instructor in mathematics and mechanical drawing in the New York State Teachers College at Albany.

MR. K. P. WILLIAMS has been appointed instructor in mathematics in Indiana University.

PROFESSOR JAMES MACMAHON, of Cornell University, will be absent on leave during the second half of the present academic year.

THE Rev. Dr. W. W. SKEAT, professor of Anglo-Saxon in Cambridge University, died October 7, at the age of 76 years. Before taking up his studies in philology, Professor Skeat was for some years lecturer in mathematics in Christ's College, Cambridge University.

THE death is announced of Professor P. TREUTLEIN, of the Goetheschule of Karlsruhe, at the age of 67 years.

DR. GASTON COMBEBIAC died at Limoges July 12 at the age of 50 years.

DR. J. M. VAN VLECK, professor of mathematics in Wesleyan University from 1853 to his retirement as emeritus professor in 1904, died on November 4, aged 79 years. Professor Van Vleck had been a member of the American Mathematical Society since 1891 and was Vice-President of the Society in 1903–1904.

CATALOGUES of mathematical books: Galloway and Porter, 30 Sidney Street, Cambridge, England, catalogue 58, about 450 titles, ancient and modern.—Bowes and Bowes, 1 Trinity Street, Cambridge, England, catalogue 362, 1791 titles, mostly before 1800.—G. Fock, Schlossgasse 7, Leipzig, catalogue 429, about 4700 titles.—B. Liebisch, Kurprinzstrasse 6, Leipzig, catalogue 588, about 2000 titles.

NEW PUBLICATIONS.

I. HIGHER MATHEMATICS.

ABHANDLUNGEN zur Sammlung mathematischer Modelle. Herausgegeben von H. Wiener. 2tes Heft. Leipzig, Teubner, 1911. M. 0.80

ALBERTI D'ENNO (J. GRAF). Die Lösung des grossen Fermatschen Problems. Trient, Moncher, 1911.

ARCHIMEDES, The method of. Recently discovered by Heiberg. Edited by Sir T. L. Heath. Cambridge, University Press, 1912. 8vo. 51 pp. 2s. 6d.

BERGHOLZ (O. A.). Die Lösung des Fermatschen Problems $x^n + y^n = z^n$. Dessau, H. S. Art'l, 1912. M. 1.00

——. Kennzeichnung der n-Potenz-Differenzen als Impotenzen. Dessau, H. S. Art'l, 1912. 32 pp. M. 1.50

BLUMBERG (H.). Ueber algebraische Eigenschaften von linearen homogenen Differentialausdrücken. (Diss.) Göttingen, 1912.

BRENKE (W. C.). See DAVIS (E. W.).

BRUNSCHVICG (L.). Les étapes de la philosophie mathématique. (Bibliotèque de philosophie contemporaine.) Paris, Alcan, 1912. 8vo. 11+592 pp. Fr. 10.00

BÜCHER, Neue, über Naturwissenschaften und Mathematik. (Die Neuigkeiten des deutschen Buchhandels, nach Wissenschaften geordnet.) Mitgeteilt Sommer 1912. Leipzig, Hinrichs. 8vo. Pp. 23–46. M. 0.30

BURKHARDT (H.). Funktionentheoretische Vorlesungen. 1ter Band, 2tes Heft. Einführung in die Theorie der analytischen Funktionen einer komplexen Veränderlichen. 4te, durchgesehene Auflage. Leipzig, Veit, 1912. 8vo. 12+262 pp. Cloth. M. 8.00

CONWAY (A. W.). Application of quaternions to some recent developments of electrical theory. Dublin, 1911. 9d.

DAVIS (E. W.) and BRENKE (W. C.). The calculus. Edited by E. R. Hedrick. New York, Macmillan, 1912. 21+383+63 pp. Semi-flexible cloth. $2.00

DZIWINSKI (P.). Wykladi matematyki (Lectures on mathematics). 2 volumes. Cracow, 1911. M. 30.00

ECKELHARDT (F.). Differential- und Integralrechnung in leichtfasslicher Darstellung. Wien, Pichler, 1912. 8vo. 4+47 pp. M. 0.85

ELLIS (H. D.). Poems mathematical and miscellaneous. London, Chiswick Press, 1912. 16mo. 61 pp. 1s. 6d.

ENCYCLOPÉDIE des sciences mathématiques. Édition française. Tome I, volume 2, fascicule 4: Théorie des formes et des invariants, par F. W. Meyer et J. Drach. Leipzig, Teubner, 1912. 8vo. Pp. 425–520. M. 3.60

——. Edition française. Tome II, volume 1: Fonctions des variables réelles. Fascicule 2: Recherches contemporaines sur la théorie des fonctions, par L. Zoretti, P. Montel et M. Fréchet.—Calcul différentiel, par A. Voss et J. Molk. Leipzig, Teubner, 1912. 8vo. Pp. 113–336. M. 8.40

EULER (L.). Opera omnia. Series II: Opera mechanica et astronomica. Volumina I et II: Mechanica sive motus scientia analytice exposita. Ed. P. Stäckel. Adjecta est Euleri effigies ad imaginem a Webero aeri incisam expressa. Leipzig, Teubner, 1912. 8vo. 14+407+460 pp. M. 56.00

FEHR (H.). Enquête de l'Enseignement Mathématique sur la méthode de travail des mathématiciens. Publié avec la collaboration de T. Flournoy et E. Claparède. 2e édition, conforme à la première, suivie d'une note sur l'invention mathématique par H. Poincaré. Paris, Gauthier-Villars, 1912. 8vo. 8+137 pp. Fr. 5.00

FELTES (J. J.). Verhandlung der allgemeinen Auflösung des Theorems Fermats. Den Haag, van Dorp, 1912. 8vo. 3+49 pp. M. 3.00

FIEDLER (R.). See JORDAN (C.).

FINZEL (A.). Die Lehre vom Flächeninhalt in der allgemeinen Geometrie. Leipzig, Teubner, 1912. 8vo. 38 pp. M. 1.20

FORSYTH (A. R.). Lectures on the differential geometry of curves and surfaces. Cambridge, University Press, 1912. 4to. 23+525 pp. $7.00

GAUTIER (D.). Mesure des angles. Hyperboles étoilées et développantes. Paris, Gauthier-Villars, 1912. 8vo. 4+84 pp. Fr. 2.00

GOLDSCHMIDT (E.). Ueber die numerische Verwendbarkeit der Methoden zur Auflösung unendlich vieler linearer Gleichungen. (Diss.) Würzburg, 1912.

GRIMM (C.). Einiges über Potenzzahlen. Frankfurt, Knauer, 1912. 8vo. 12 pp. M. 0.60

GUIOT (J.). Le calcul vectoriel et ses applications à la géométrie réglée. Paris, Hermann, 1912. 8vo. 128 pp.

HEATH (SIR T. L.). See ARCHIMEDES.

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INTERNATIONAL catalogue of scientific literature. Tenth annual issue (June, 1910–June, 1911). A: Mathematics. London, Harrison, 1912. 8vo. 8+248 pp. 15s.

JORDAN (C.) et FIEDLER (R.). Contribution à l'étude des courbes convexes fermées et de certaines courbes qui s'y rattachent. Paris, Hermann, 1912. 8vo. 72 pp. Fr. 3.00

KARPINSKI (L. C.). See SMITH (D. E.).

KEMPNER (A. J.). Ueber das Waringsche Problem und einige Verallgemeinerungen. (Diss.) Göttingen, 1912.

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KURRAS (K.). Repetitorium der gesammten Geometrie, einschliesslich der Differential- und Integralrechnung. Berlin, Ulrich, 1912. 8vo. 112 pp. M. 3.50

LEBON (E.). Henri Poincaré. Biographie. Bibliographie analytique des écrits. 2e édition, entièrement refondue. (Savants du jour.) Paris, Gauthier-Villars, 1912. 8vo. 4+112 pp. Fr. 7.00

LETZ (E.). Die Verfolgungskurve des Kehlkreises auf den Rotationsflächen constanter Krümmung. (Diss.) Halle a. S., 1912.

LIEBMANN (H.). Nichteuklidische Geometrie. 2te, neubearbeitete Auflage. (Sammlung Schubert, Nr. 49.) Berlin, Göschen, 1912. 8vo. 6+222 pp. Cloth. M. 6.50

LÖFLUND (F.). Ueber algebraische Raumkurven. (Diss.) Tübingen, 1912.

LOTS (A.). Der Beweis des grossen Fermatschen Satzes. Altenburg, 1912.

LUSSAN (E.). Deuxième essai de démonstration du théorème de Fermat. Paris, Gauthier-Villars, 1912. 8vo. 9 pp. Fr. 1.00

LYLE (J. N.). The euclidean or common sense theory of space. Bentonville, Ark., Lyle, 1912. 16mo. 63 pp. $0.60

MAIRE (A.). L'oeuvres scientifiques de Blaise Pascal. Avec portrait de Pascal. Bibliographie. Critique et analyse de tous les travaux qui s'y rapportent. Avec préface de P. Duhem. Paris, Hermann, 1911. 8vo. 20+200 pp. Fr. 15.00

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MEYER (W. F.). Differential- und Integralrechnung. 1ter Band: Differentialrechnung. 2te, durchgesehene und erweiterte Auflage. (Sammlung Schubert, Nr. 10.) Berlin, Göschen, 1912. 8vo. 15+418 pp. Cloth. M. 9.00

MITTENZWEY (L.). Mathematische Kurzweil. Aufgaben, Kunststücke, geistanregende Spiele, verfängliche Schlüsse, Scherze, und Ueberraschungen aus der Zahlen- und Formenlehre. 6te Auflage. Leipzig, Klinkhardt, 1912 8vo. 103 pp. M. 1.60

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NIELSEN (N.). Beretning om den anden Skandinaviske Matematikerkongres i Kjöbenhavn 1911. Kjöbenhavn, Gyldendalske Boghandel, 1912. 8vo. 16+192 pp. Kr. 6.00

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PICARD (E.). Oeuvres de Charles Hermite, publiées sous les auspices de l'Académie des Sciences. Tome III, avec un portrait. Paris, Gauthier-Villars. 8vo. 6+524 pp. Fr. 18.00

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RAMEL (G. F.). Die Lösung des sogenannten Fermatschen Beweisproblems. Zürich, Müller, 1912. 8vo. 27 pp. M. 1.00

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VERZEICHNIS mathematischer Modelle. Sammlungen von H. Wiener und P. Treutlein. Leipzig, Teubner, 1912.

WANG (K. T.). The differentiation of quaternion functions. Dublin, 1911. 9s

WATSON (G. N.). Theory of asymptotic series. London, 1911. 4to. 2s.

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ZACHARIAS (M.). Einführung in die projektive Geometrie. (Mathematische Bibliothek, Nr. 6.) Leipzig, Teubner, 1912. 8vo. 4+51 pp. M. 0.80

II. ELEMENTARY MATHEMATICS.

BOURLET (C.). Algèbre. Paris, Hachette, 1912. 16mo. 323 pp. Fr. 3.00

——. Eléments de géométrie. 3e édition revue. Paris, Hachette, 1912. 16mo. 6+383 pp. Fr. 2.50

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RÉBOUIS (A. G.). See CAMMAN (P.).

RECKNAGEL (G.) und WETZSTERN (G.). Ebene Geometrie mit vorbereitender Raumlehre. 2te, verbesserte Auflage. Nürnberg, Koch, 1912. 8vo. 15+213 pp. Cloth. M. 2.50

ROBBINS (E. R.). Plane trigonometry. New York, American Book Co. 8vo. 153 pp. Cloth. $0.60

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SCHMALZ (K.). Schul-Mathematik in Tabellen, Disposition und Repetitorium. Halle, Waisenhaus, 1912. 8vo. 36 pp. M. 0.75

SCHNELL (H.). Elementare Ableitung der Pendelformel $t = \pi\sqrt{l/g}$. Vereinfachte Berechnung der Zahl π und kleine Beiträge zur Schulmathematik. (Progr.) Landsberg a. L., 1912. 8vo. 48 pp.

SCHULTZ (E.). Elemente der ebenen Geometrie auf funktionaler Grundlage. Berlin, Salle, 1912. 8vo. 4+116 pp. M. 1.80

SCHWAB (K.) und LESSER (O.). Mathematisches Unterrichtswerk. 2ter Band: Geometrie. Ausgabe A. 3 Teile. Leipzig, Freytag, 1912. 8vo. 290+140+120 pp. Cloth. M. 7.30

——. 3ter Band, 1ter Teil: Synthetische Geometrie der Kegelschnitte und analytische Geometrie. 2te Auflage. Leipzig, Freytag, 1912. 8vo. 208 pp. Cloth. M. 2.80

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UNTERRICHT, Der mathematische, in der Schweiz. Nr. 2, von H. Fehr, J. Stöcklin und D. Badertscher. Basel, Georg, 1912. 8vo. 16+106 pp. M. 2.25

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ZWERGER (M.). Ebene und sphärische Trigonometrie. 13te Auflage. München, Lindauer, 1912. 8vo. 7+60+21 pp. M. 1.60

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CLAUDEL (J.). Introduction à la science de l'ingénieur. Partie théorique. 8e édition, entièrement refondue. Tome I: Arithmétique, algèbre, géométrie, trigonométrie, topographie. Paris, Dunod et Pinat, 1912. 8vo. 7+955 pp.

COBB (H. E.). Elements of applied mathematics. Boston, Ginn, 1911. 12mo. 8+274 pp. Cloth. $1.00

EBERHARDT (C.). Theorie und Berechnung von Motor-Luftschiffen. Berlin, 1912. M. 10.00

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HANDBUCH der Elektrizität und des Magnetismus. In 5 Bänden. Bearbeitet von F. Auerbach, K. Baedeker, P. Cermak, und L. Graetz. 1ter Band, 1te Lieferung. Leipzig, Barth, 1912. 8vo. Pp. 1–156. M. 6.00

HARTMANN (T.). Zur Theorie der Momentanbewegung eines ebenen veränderlichen Systems. (Diss.) Rostock, 1912.

HÄUSSLER (A.). Wegweiser für die Gravitationsforschung. Berlin, Friedländer, 1912. M. 2.00

KRYLOW (A. U.). On approximate computations. (Russian.) Petersburg, 1911.

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MANITIUS (K.). Des Claudius Ptolemäus Handbuch der Astronomie. 1ter Band. Aus dem Griechischen übersetzt und mit erläuternden Anmerkungen versehen. In 2 Bänden. Leipzig, Teubner, 1912. 8vo. 28+462 pp.

METTLER (H.). Graphische Berechnungs-Methoden. II und III: Aeromechanik. Zürich-Selnau, Leemann, 1912. 16mo. 78 and 130 pp.

MORITZ (R. E.). College engineering notebook. Boston, Ginn, 1911. 4to. In biflex binder. $1.00

MÜLLER (A.). Ueber eine Verallgemeinerung des Begriffes der Zentrifugalkraft. (Progr.) Köln, 1912.

OPITZ (H.). Ueber das erste Problem der Dioptrik. II: Materialien zu einer kritischen Geschichte des Problems. Berlin, Weidmann, 1912. M. 1.00

PEABODY (C. H.). Propellers. New York, Wiley, 1912. 8vo. 3+132 pp. Cloth. $1.25

RÖMERT (J.). Der praktische Mathematiker. 2 Hefte. Halle, 1912. 44+40 pp. Boards. M. 4.00

RYAN (W. T.). Design of electrical machinery. Volume II: Alternating current transformers. New York, Wiley, 1912. 8vo. 7+119 pp. Cloth. $1.50

SCHLOTKE (J.). Darstellende Geometrie. 3ter Teil: Perspektive. 3te, ergänzte Auflage, herausgegeben von C. Rodenberg. Leipzig, Degener, 1912. 8vo. 8+133 pp. M. 4.60

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SCHMIDT (F.). Beiträge zur Verteilung der Elektrizität auf zwei leitenden Kugeln, insbesondere für den Fall der Berührung. (Diss.) Halle a. S., 1912.

SYLLABUS of mathematics for engineering students. Prepared by a joint committee of the American Mathematical Society and the American Association for the Advancement of Science. Ithaca, N. Y., Society for the Promotion of Engineering Education, 1912. $0.75

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